EEEN203 Circuit Analysis

Transformers (Electric)

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Who am I?

- Ramesh Rayudu – BEng (India), ME (Canterbury), PhD (Canterbury)
- 15 years in NZ Electricity Industry
- 13 years in NZ Academia
- Research Interests:
  - Power System Engineering – AC, DC, Renewable Energy Engineering
  - Power Electronics
  - Geothermal Energy
  - Electricity – Hydrogen Nexus

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Transformers and Electricity

- Video of a transformer fault in USA
- Transformer faults are much more dangerous than any other power system faults
- Transformer blowouts cost millions of dollars (equipment and loss of power)
- At least 3 to 4 blowouts in NZ every year.

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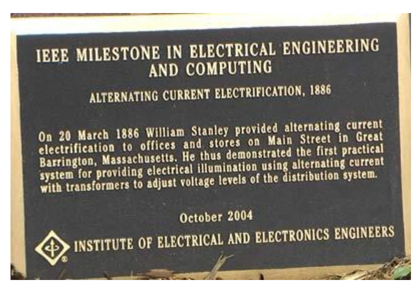
Transformers – the AC vs DC debate

- Transformers made AC usage more preferable.
- How? – by stepping up the voltage and thereby saving copper (metal)
- In non Power Electronics world (pre-1980s), they provided lower voltages for toys (non-lethal)
- Automatically regulate voltages in power systems
- A best way to match load to a source
- Highly efficient – over 95%



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In this week?

- Look at ideal transformer first - 100% efficient
- Then we will model a real transformer
- Finally, we will include a transformer in a power system and analyse

- Why is this topic first?
  - To complete the lab next week
  - We will go through the basics of AC in week 11 and 12

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Dangerous Voltages

Vector's Schools Programme

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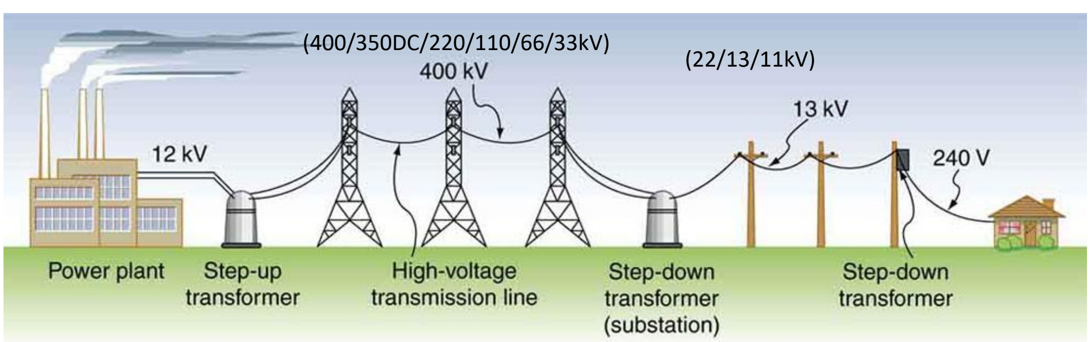
How do they run our world?



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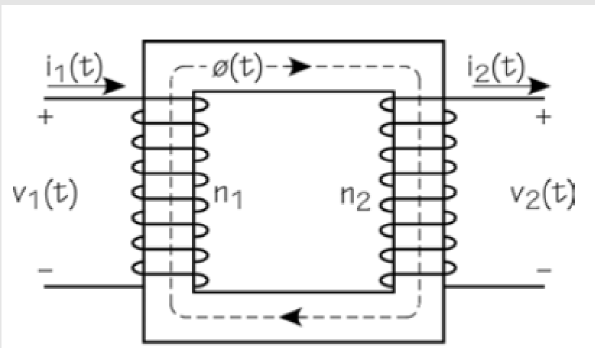
Ideal Transformers



- Wrap a couple of wires around an iron core, you have a transformer – that simple
  - Electromagnetic induction or mutual induction
- Power transfer – left is primary, right is secondary; from primary to secondary
- Power transfer depends on the number of coils – basically the turns ratio of the two coils

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How does Transformers work?

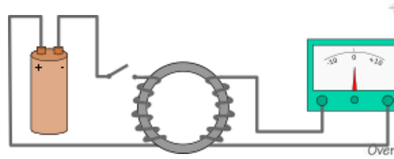
- More magnetics in EEEN313
- Faraday's laws

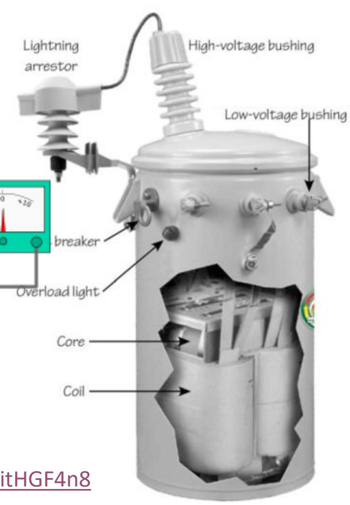
$$v(t) = \frac{d\phi(t)}{dt}$$

- Now use this for our transformer

$$v_1(t) = n_1 \frac{d\phi(t)}{dt}$$

$$v_2(t) = n_2 \frac{d\phi(t)}{dt}$$





https://www.youtube.com/watch?v=UchitHGF4n8

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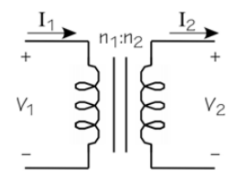
Transformers

- Since we don't have current in the previous equation, lets use Ampere's law

$$\phi(t) \propto n_1 i_1(t)$$
$$\phi(t) \propto n_2 i_2(t)$$

- Since we use AC, we need to change to phasor notation



- Makes it easy for us to deduce simple relationships

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Transformer Equations in Phasor Domain

- From Faraday's laws, we get voltage and turns ratio

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

- From Ampere's law, we get current and turns ratio

$$\frac{I_1}{I_2} = \frac{n_2}{n_1}$$

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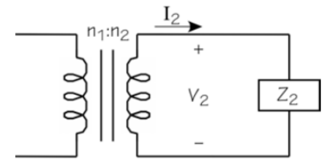
Impedance



- Suppose we connect an impedance to the load

$$Z_2 = \frac{V_2}{I_2}$$

- Replace V_2 and I_2 by their equivalents and we get

$$Z_2 = \frac{\frac{n_2}{n_1}V_1}{\frac{n_1}{n_2}I_1} = \left(\frac{n_2}{n_1}\right)^2 \frac{V_1}{I_1} = \left(\frac{n_2}{n_1}\right)^2 Z_1$$

$$Z_1 = \left(\frac{n_1}{n_2}\right)^2 Z_2$$

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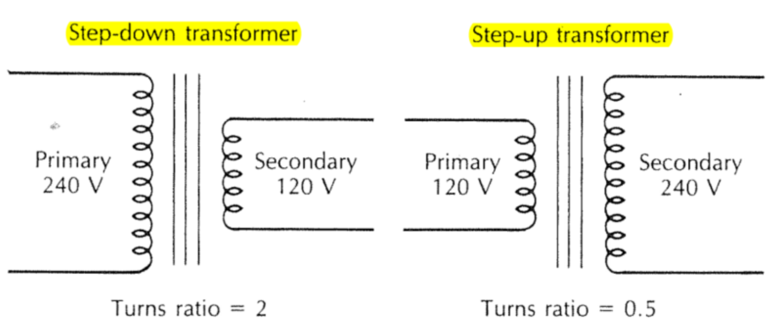
An Example



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Example 2

- A step-down transformer has a turns ratio of 4 to 1or 4. If the transformer secondary voltage is 120 V, determine the primary voltage.

$$\text{Primary voltage} = 120\ \text{V} \times 4 = 480\ \text{V}$$

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Ratings of a Transformer

- Transformers are rated in volt-amps (VA) or (kilo) kVA
- VA ratings are used because the rating of a transformer depends on losses and losses depend on V and I.
- Ratings mean primary and secondary are designed to withstand the VI (VA or kVA) rating.

$$\text{Full-load current} = \frac{\text{VA rating}}{\text{Voltage}}$$

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Example

- A single-phase transformer with a 2-kVA rating has a 480-V primary, and a 120-V secondary. Determine the primary and secondary full-load currents of the transformer.

Primary full-load current

$$= \frac{2\text{ kVA} \times 1\,000}{480\text{ V}} = 4.17\text{ A}$$

Secondary full-load current

$$= \frac{2\text{ kVA} \times 1\,000}{120\text{ V}} = 16.67\text{ A}$$

- Note: Transformer current will be higher in the winding that produces low voltage (saving of copper)
- Homework: Given secondary current, how do you get primary full load current?

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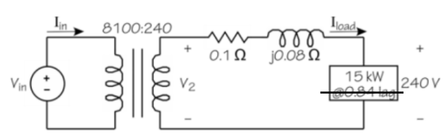
Example 1



We need to supply a 15 kW load at 240V. Find the supply side voltage and current. **Assume unity power factor**.

$$I_{LOAD} = \frac{15000}{240 \times PF(1)} = 62.5\text{ A}$$

V_2 = load voltage + V_{drop} across the Z_{line}

$$V_2 = 240 + (0.1 + j0.08) \times 62.5$$

$$V_{in} = \frac{8100}{240} \times V_2$$

$$I_{in} = \frac{240}{8100} \times I_{LOAD}$$

Powe delivered by generator

$$P_{in} = |V_{in}||I_{in}|$$

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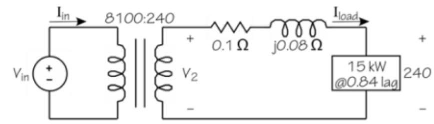
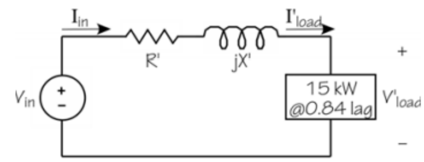
'Reflected' through transformer



- We know that voltage goes through by the turns ratio, current by the inverse of the ratio and impedance by ratio squared.
- Since power is conserved, we can 'reflect' the load through the transformer.



$$V'_{load} = \frac{8100}{240} 240 = 8100 \text{ V}$$

$$R' = \left(\frac{8100}{240}\right)^2 0.1 = 113.9 \ \Omega$$

$$X' = \left(\frac{8100}{240}\right)^2 0.08 = 91.1 \ \Omega$$

$$I'_{load} = \frac{15000}{(8100)(0.84)} \angle -\cos^{-1} 0.84 = 2.205\angle -32.9° \text{ A}$$

$$V_{in} = 8100 + (113.9 + j91.1) 2.205\angle -32.9°$$

$$= 8100 + 321.6\angle 5.8° = 8420\angle 0.2° \text{ V}$$

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Real Transformer

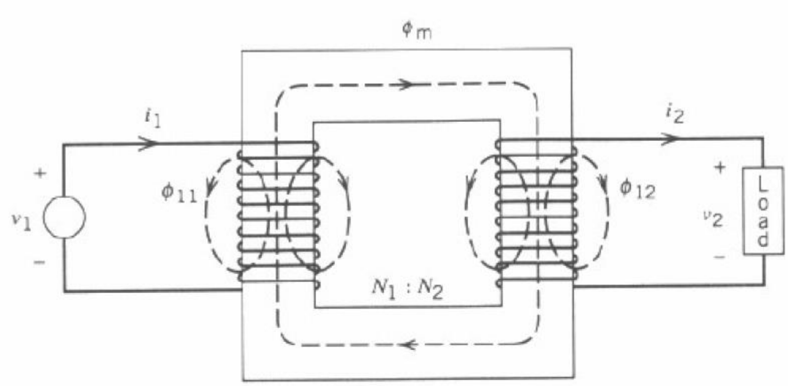

FIGURE 4.10 An actual transformer.

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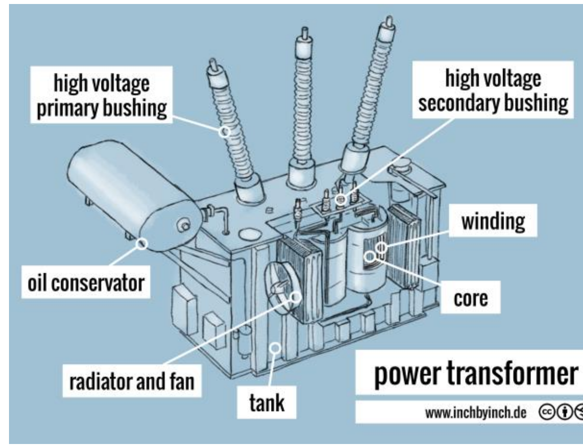
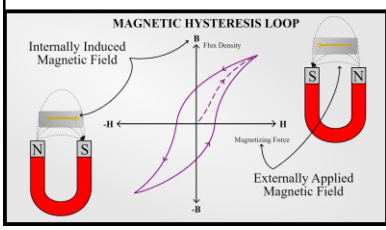
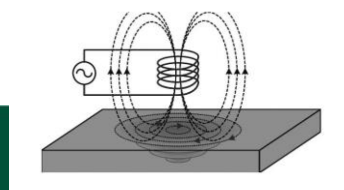
Real Transformers

- Have resistance in the windings
- Not all of the flux produced by one winding links with the other (flux leakage)
- Magnetic core has finite permeability
- Core losses
  - Hysteresis
  - Eddy currents







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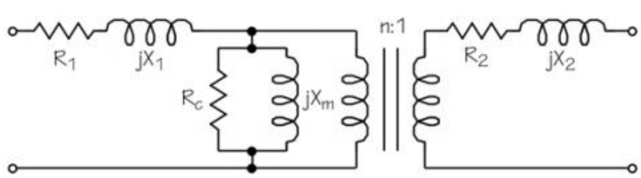
Real Transformers

- Transformers are made of wires and each wire has a resistance
- Since Transformer coils look like inductors, there would be some inductance too

X_1 and X_2 are leakage reactance
R_c is core loss
X_m is magnetic reactance



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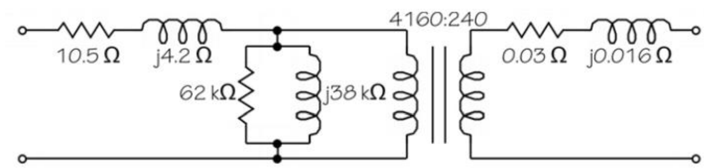
Model with real numbers

- From a 20 kVA, 4160-240 V transformer



- Wire resistances are minimised; we want low core losses
- Note the series elements are small and parallel ones are large
- Moving the parallel elements to the right of series elements- less effect on the main current flowing through the series elements.

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Transformer Losses

Transformer losses consist of:

- Copper losses in the windings
  - Depend on load current
- Hysteresis and eddy-current losses in the core
  - Constant for constant flux (constant voltage) conditions
- Stray losses due to currents induced by leakage fluxes in the transformer structure
  - Negligible for a well-designed transformer

The maximum total losses in a transformer account to < 3%

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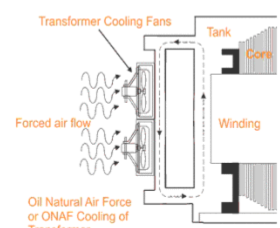
Transformer Rating

Transformer ratings are provided to keep the operating temperature within acceptable limits. A transformer's rating is based upon the following:

- Nominal current
  - To limit copper losses
- Nominal voltage and frequency
  - To limit core losses
  - Transformer size based upon flux density limit in core material
- Apparent power rating
  - Based on product of nominal current and nominal voltage
  - A transformer can become fully loaded at sufficient levels of reactive power, even if no real power is being delivered.
- Cooling

Transformer Cooling

- Cooling of a transformer increases the rate of heat dissipation and hence improves the transformer rating:
- Low-voltage indoor transformers (<200kVA) can be passively air-cooled via natural convection
- Relative to air, oil is a better thermal conductor and electrical insulator, so it is invariably used for cooling of high-voltage, high-power transformers.
- As power rating increases, radiators, heat exchangers and forced oil/air circulation may be added to improve power dissipation

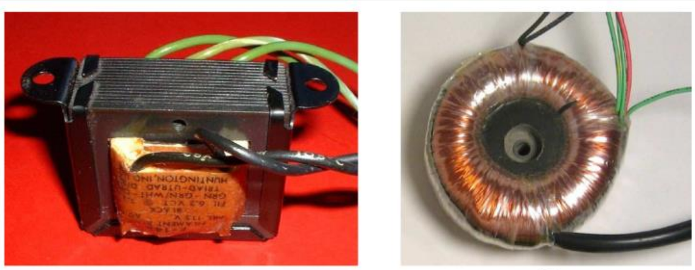


Transformer Construction

Power transformers are designed such that their characteristics approach the ideal:

- To attain high permeability, cores are made of iron-based materials
- To minimise core losses, core is laminated from high-resistivity, high-grade silicon steels
- Leakage reactances are minimised by co-winding of the coils
- Geometries are optimised to minimise turn lengths, maximise core window areas and achieve highest power densities

Types



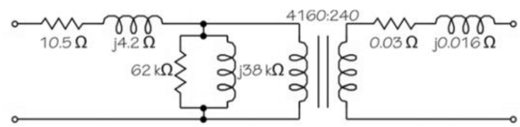
(a) (b)

Figure 64: E-I (a) and toroidal (b) transformers.

- E-I has E shaped steel capped with I shaped pieces
- Toroidal are built around a ring shaped core of silicon steel
  - Eliminates air gaps
  - Better than E-I in several ways
  - Can be used at higher frequencies in SMPS

Transformer Testing

- So how do we get the real transformer numbers stated before?
- Two basic transformer tests:
  - Open circuit test
  - Short circuit test
- All measure the current, voltage and power



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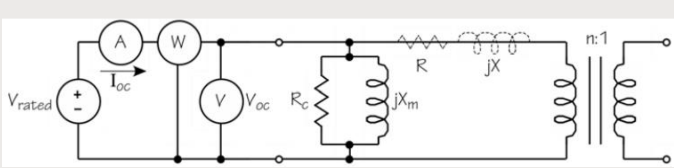
Open-circuit test

- Tells us how the transformer behaves under the rated voltage
- Ammeter, Voltage and Wattmeter measure I_{oc}, V_{oc}, P_{oc}
- Note, no load connected – secondary is left open
- So the current is very less – series elements have less effect



$$P = |V||I| \cos(\theta_v - \theta_i)$$

$$\angle I_{oc} = -\cos^{-1} \frac{P_{oc}}{|V_{oc}||I_{oc}|}$$

$$I_{oc} = |I_{oc}| \angle I_{oc} = I_c - jI_m$$

We know the voltage in the parallel elements,
So we can find the impedances

The two parallel elements are:

$$R_c = \frac{|V_{oc}|}{I_c}, \quad X_m = \frac{|V_{oc}|}{I_m}$$

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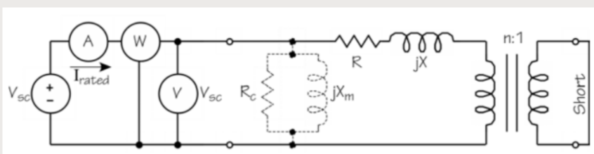
Short-circuit Test

- This is rated current test – so we short the secondary – wiring the terminals together.
- We apply a small voltage to the primary, just enough to drive the rated current into the transformer.
- So the parallel elements have small effect.
- Series elements carry the rated current.



Phasor Currents:

$$\angle I_{sc} = -\cos^{-1}\frac{P_{sc}}{|V_{sc}||I_{sc}|}$$

$$I_{sc} = |I_{sc}|\angle I_{sc}$$

Impedance of series elements:

$$R + jX = \frac{|V_{sc}|}{I_{sc}}$$

Real and imaginary parts of impedance:

$$R = \mathrm{Re}\left[\frac{|V_{sc}|}{I_{sc}}\right], \quad X = \mathrm{Im}\left[\frac{|V_{sc}|}{I_{sc}}\right]$$

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Example: Transformer Test

- Transformer is a 20-kVA, 4160:240 V
- Rated current for short-circuit test is 20 kVA/ 4160 = 4.81A
- Find the transformer parameters

$$\mathrm{V}_{oc} = 4160\mathrm{V} \qquad \mathrm{V}_{sc} = 121\mathrm{V}$$
$$\mathrm{I}_{oc} = 0.155\mathrm{A} \qquad \mathrm{I}_{sc} = 4.81\mathrm{A}$$
$$\mathrm{P}_{oc} = 292\mathrm{W} \qquad \mathrm{P}_{sc} = 509\mathrm{W}$$

We will revisit this after our lectures on 'Power' this week.

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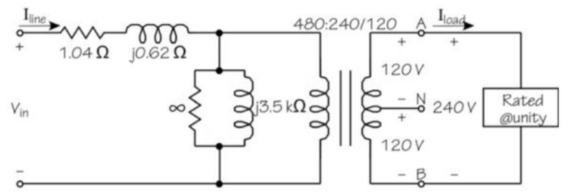
Load at Unity Power Factor - Example

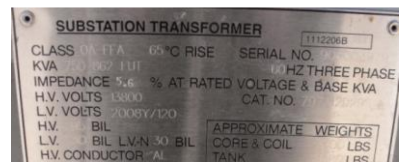
A 10 kVA, 480:240/120 V supplies 240 V at rated load
Unity Power Factor
Find voltage regulation in percentage.
Rated Load? – means we have to supply 10 kVA no matter what the power factor is!



- If the transformer rating is 10 kVA and it's supplying rated load, that rating is the same on both sides of the transformer.
- Therefore, I can calculate rated current on the primary side by dividing total power by the primary voltage:

$$I_{rated} = \frac{10\times10^3}{480} = 20.8 \text{ A}$$

$$V_{in} = 480 + (1.04 + j0.62)\,20.8\angle0^\circ$$
$$= 480 + 25.2\angle30.8^\circ = 501.8\angle1.5^\circ \text{ V}$$



$$\%VR = 100\frac{501.8 - 480}{480} = 4.5\%$$

Good! < 5%

Load at Unity Power Factor - Example

- The only loss in the transformer is the series resistance – wire heating
- Parallel branch has only inductance; resistance is very large (ignore)
- Losses

$$P_{losses} = 1.04\times20.8^2 = 450 \text{ W}$$

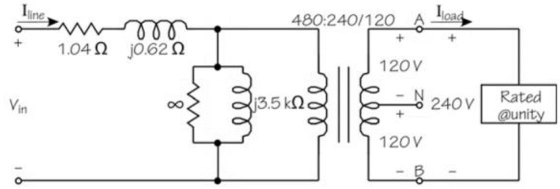
- Efficiency

$$\%\eta = 100\frac{10}{10+0.450} = 95.7\%$$

Good Again!

- But what happens if we have a poor power factor? – very common now
- We will look at this when we discuss Power Factor

Peculiar Transformer



- Notice extra terminal on the secondary
- This is called center (US spelling)-tapped transformer
- This is a common configuration in USA to supply homes
- A-N or B-N is 120 V; A-B is 240 V
- Each home there has a Centre tapped Transformer; 120V supplies appliances and lights, 240 V supplies AC, water heater, kitchen range (big loads)
- Both sides on the Transformer in the lab is centre tapped (basically taking output at half the number of coils)

Summary

- Transformers are simple elements that keep our mobiles phones and game consoles on (and also working) all the time (and also transform electricity)
- Are bidirectional
- Voltage go through turns ratio; current as inverse; impedance as the square.
- Power is conserved
- Real transformer has series and parallel resistance and reactance
- Their values can be determined by open and short circuit tests