Parametric Channel Prediction for Narrowband Mobile MIMO Systems Using Spatio-Temporal Correlation Analysis

Ramoni O. Adeogun, Paul D. Teal and Pawel A. Dmochowski
School of Engineering and Computer Science
Victoria University of Wellington,
Wellington New Zealand
E-mail:(ramoni.adeogun,paul.teal,pawel.dmochowski)@ecs.vuw.ac.nz
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Abstract

In this paper, we propose an ESPRIT-based parametric prediction scheme for narrowband MIMO systems that fully exploits both temporal and spatial correlations in realistic MIMO channels. The proposed predictor uses a vector transmit spatial signature model and two-dimensional ESPRIT for the estimation of the channel parameters. The proposed scheme outperforms existing algorithms and is well suited to both two dimensional azimuth only and three dimensional MIMO spatial channel models.


1 Introduction

In mobile MIMO wireless communication systems, full multiplexing and/or diversity gain can be obtained when channel state information (CSI) is available at the transmitter. Channel reciprocity is often exploited in time division duplex (TDD) systems to obtain CSIT. However, in frequency division duplex (FDD), CSI is estimated at the receiver and fed back to the transmitter [7]. Because of delays in processing and feedback that is inherent in practical systems, such state information rapidly becomes outdated before its actual usage for precoding and/or link adaptation at the transmitter. This results in performance degradation and a reduction in the gains expected from using MIMO under time varying channel conditions. Prediction of the channel into the future has been recognised has an effective means of mitigating the performance degradation due to feed back delays.

The problem of multipath prediction has been well addressed for single input single output (SISO) systems. In [4, 5] the narrowband SISO channel is modelled as an autoregressive (AR) process of a particular order and a linear predictor that minimizes the mean squared error (MSE) is used to predict future states of the channel using the AR parameters. These schemes consider the time-varying channel as a stochastic wide sense stationary process (WSS) and use the temporal correlation for prediction without accounting for the physical scattering phenomenon causing the fading. Other researchers [2, 16] have used the ray based sum of sinusoids approach where the fading channel is modelled as a sum of a finite number
of plane waves. Direction of arrival estimation algorithms are used to estimate the number of scattering sources and angles of arrival and the channel is predicted using the model. Analytical and simulation results on SISO prediction have proven that with dense scattering, SISO channels can only be predicted over a very short distance (of the order of tenths of a wavelength) depending on the environment and propagation scenarios. The bound on SISO channel prediction error in [14] indicated that channel prediction schemes require channel state information over several wavelengths in order to accurately predict the channel and that prediction beyond a wavelength is not realistic particularly in practical cases where the stationarity assumption does not hold long enough relative to the length of observation.

The possibility of predicting multi-antenna channels was first investigated in [3] through an evaluation of downlink beamforming with channel prediction. It was shown that channel prediction improves MISO smart antenna system performance. An explanation for this is that more structure of the wavefield is revealed through multiple sampling and better prediction can therefore be expected. Bounds on the prediction error of MIMO channels [13] indicate that better prediction can be obtained by utilizing the spatial parameters of the MIMO channel. The authors used auto-regressive (AR) modelling for the prediction of a beamspace transformed CSI and an inverse transformation was performed on the predicted CSI. They argue that the transformation reduces the effective number of rays present in the channel which ultimately results in longer prediction. The proposed beamspace transformation matrix is, however, not well conditioned and so the inverse transformation is difficult to perform. A similar approach based on ray cancelling was presented in [10]. The scheme utilizes QR decomposition to overcome the ill-conditioning of the transformation matrix. It was however assumed that the angle of arrival and angle of departure are known. Estimation of the angle of departure require the transmitter to have a large number of transmit antennas which is not realistic in practical systems and moreover the error propagation problem with AR model based prediction limits the application in systems where long range prediction is required.

The novel contributions of this paper are as follows.

- We propose a parametric prediction scheme for narrowband MIMO systems that fully exploits both the temporal and spatial correlations in realistic spatial channel models. By utilizing the spatial correlation of the channel, we reduce the number of parameters estimated, improve estimation accuracy and increase the achievable prediction horizon.

- We propose an ESPRIT-based approach for AOA estimation that utilizes spatial correlation to achieve two-dimensional angle estimation using a one dimensional array at right angle to the direction of motion.

- We perform rigorous simulations to evaluate the performance of the proposed algorithm and compare it with existing schemes. We analyze the effects of the number of antennas, SNR, and training length on MIMO prediction performance. Simulation results show that the proposed algorithm outperforms existing algorithms.

2 Propagation Models

2.1 System Model

We consider a narrowband MIMO System with $N_T$ transmit and $N_R$ receive antennas. The received signal is given by

$$y(t) = H(t)x(t) + w(t)$$  \hspace{1cm} (1)

where $x(t) = [x_1(t), \ldots, x_{N_T}(t)]^T$ is the $N_T \times 1$ vector of transmitted signals, $y(t) = [y_1(t), \ldots, y_{N_R}(t)]^T$ is the $N_R \times 1$ vector of received signals, $w(t) = [w_1(t), \ldots, w_{N_R}(t)]^T$ is the $N_R \times 1$ vector of received signal noise and $H(t)$ is the $N_R \times N_T$ channel impulse response matrix. We here assume that interference can be modelled as a noise component and that estimates of the MIMO channel impulse response can be obtained by transmitting known
training signal, but the received signal noise introduces imperfections in this estimation. The channel estimate is therefore modelled as a summation of the actual channel and some error

\[ \hat{H}(k) = H(k) + N(k); \quad k = 1, 2, \cdots, K \] (2)

where \( K \) denotes the number of available CSI estimates, and \( N(k) \) is the \( N_R \times N_T \) matrix of estimation noise at the \( k \)th time instant. We simplify this estimation by assuming \( N(k) \) to be spatially and temporally white complex Gaussian random variable with zero mean and variance \( \sigma_N^2 \).

2.2 Narrowband MIMO Channel Model

A commonly used multipath model is the ray based sum of sinusoids model. The model is defined for a single input single output (SISO) system as the superposition of \( P \) scattering sources

\[ h(t) = \sum_{p=1}^{P} \alpha_p \exp(j\omega_p t) \] (3)

where \( \alpha_p \) is the complex amplitude of the \( p \)th scattering source and \( \omega_p \) is the Doppler frequency defined as

\[ \omega_p = kV \cos \theta_p \] (4)

In (4), \( V \) is the mobile velocity, \( \theta_p \) is the angle of arrival of the \( p \)th scatterer, \( k = \frac{2\pi}{\lambda} \) is the wave number and \( \lambda \) is the wavelength. The SISO model in (3) can be extended to modelling of a MIMO propagation channel via the introduction of the spatial dimension [13]

\[ H(t) = \sum_{p=1}^{P} \alpha_p b_r(\theta_p) b_t^T(\phi_p) \exp(j\omega_p t) \] (5)

where \( \phi_p \) is the angle of departure and \( b_r \) and \( b_t \) are the receive and transmit array response vectors in the direction of the \( p \)th scattering source, respectively. For a uniform linear array (ULA) with antenna spacing \( \Delta_r \) at the receiver, the receive array steering vector is defined as

\[ b_r(\theta_p) = [1, \exp(j\Omega_p), \cdots, \exp(j(N_R-1)\Omega_p)]^T \] (6)

where \( \Omega_p = k\Delta_r \sin \theta_p \). The transmit response vector, \( b_t(\phi_p) \), is obtained by replacing \( \Delta_r \) with \( \Delta_t \) and \( \theta_p \) with \( \phi_p \) in (6). The two dimensional model in (5) is based on the assumption that the effects of the elevation spectrum can be neglected. Recent studies in MIMO channel modelling have however shown that the elevation spectrum needs to be accounted for, particularly in indoor and in-vehicle outdoor mobile scenarios where reflections from the ceiling and/or ground are significant [12, 17]. Thus, we introduce elevation angles of arrival and departure in (5) to obtain

\[ H(t) = \sum_{p=1}^{P} \alpha_p b_r(\theta_p, \zeta_p) b_t^T(\phi_p, \varepsilon_p) \exp(j\omega_p t) \] (7)

where \( \zeta_p \) and \( \varepsilon_p \) are the elevation angle of arrival and departure, respectively. The steering vectors, \( b_r \) and \( b_t \) for the 3D model are of the form

\[ b_r(\theta, \varphi) = [1, \exp(j\Phi), \exp(j2\Phi), \cdots, \exp(j(N_R-1)\Phi)]^T \] (8)

where \( \Phi = k\Delta_r \sin \theta \sin \zeta \) and the Doppler frequency is \( \omega = kV \sin \theta \cos \zeta \).
3 Prediction Scheme

Prediction of the MIMO channel impulse response using the 2D model in (6) and 3D model in (8) is essentially a model parameter estimation problem. This implies that for our prediction, we need to estimate the AOD, AOA and complex amplitudes of the contributing rays. Since the transmitter is stationary, estimation of the AOD requires a large number of transmit antennas to be deployed at the transmitter. In order to overcome this problem, we reduce the channel models to

\[ H(t) = \sum_{p=1}^{P} b_r(\theta_p) v_p^T \exp(j\omega_p t) \]  

and

\[ H(t) = \sum_{p=1}^{P} b_r(\theta_p, \zeta_p) v_p^T \exp(j\omega_p t) \]

where \( v_p \) is an \( 1 \times N_T \) vector defined as the product of the complex amplitudes and transmit array steering vector. We henceforth refer to \( v_p \) as the transmit spatial signature (TSS). We now discuss the procedure for the parameter estimation stage of the proposed ESPRIT-based MIMO prediction (ESMIP) algorithm.

3.1 Covariance Matrix and Subspace Dimension Estimation

Using the \( K \) available channel estimates, we form an \( N_R Q \times N_T L \) block-Hankel matrix

\[ \hat{D} = \begin{bmatrix} \hat{H}(1) & \hat{H}(2) & \cdots & \hat{H}(S) \\ \hat{H}(2) & \hat{H}(3) & \cdots & \hat{H}(S+1) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{H}(Q) & \hat{H}(2) & \cdots & \hat{H}(K) \end{bmatrix} \]  

where \( S = (K - Q + 1) \) is the number of averages of correlation estimates and \( Q \) is the Hankel matrix size which defines the number of correlation lags and the size of the covariance matrix. The spatio-temporal correlation matrix is then estimated from \( \hat{D} \) as

\[ \hat{R} = \hat{D} \hat{D}^\dagger / (N_T S) \]

where \( \dagger \) denotes the Hermitian conjugate transpose. Let \( \hat{\lambda}_1 \geq \cdots \geq \hat{\lambda}_{N_R Q} \) be the eigenvalues of \( \hat{R} \) in descending order of magnitude. The number of dominant sources can be estimated using the minimum description length (MDL) as [6]

\[ \hat{P} = \arg \min_{z=1, \cdots, N_R Q-1} \left[ S \log(\lambda_z) + \frac{1}{2} (z^2 + z) \log S \right] \]

3.2 DOA/Receive Steering Vector Estimation

3.2.1 Data Model

In order to estimate the AOA from the spatio-temporal correlation matrix in (12), we derive a model for the data in \( \hat{D} \). Let \( d(\theta_p, \zeta_p) \) be a \( Q \times 1 \) vector defined as

\[ d^m(\theta_p, \zeta_p) = [\exp(jm\gamma_p), \exp(j(m + 1)\gamma_p), \cdots, \exp(j\nu\gamma_p)]^T \]

where \( \gamma_p = 2\pi \Delta x \cos(\theta_p) \sin(\zeta_p) \) is the normalized Doppler frequency, \( \Delta x \) is the spatial sampling interval and \( \nu = m + Q - 1 \). Each group of \( N_T \) columns of \( \hat{D} \) corresponds to \( N_T \) independent
observations at the same sampling instant. The columns of $D$ can therefore be modelled as

$$
\mathcal{H}(u) = \sum_{p=1}^{N} v_p(i) b_r(\theta_p, \varphi_p) \otimes d^m(\theta_p, \varphi_p)
$$

(15)

where $u = mN_T + i$, $i = 1, \ldots, N_T$ and $m = 0, 1, \ldots, P-1$ and $\otimes$ denotes the Kronecker product. A matrix representation of (15) is

$$
\mathcal{H}(u) = A(\theta, \varphi)\beta(u) + n
$$

(16)

where $A(\theta, \varphi) = [b_r(\theta_1, \zeta_1) \otimes d^1(\theta_1, \zeta_1), \ldots, b_r(\theta_N, \zeta_N) \otimes d^1(\theta_N, \zeta_N)]$ is an $N_pQ \times N$ matrix equivalent to the array steering matrix of a two dimensional array and $\beta(u) = [v_1(i) \exp(j(m-1)\gamma_1), \ldots, v_N(i) \exp(j(m-1)\gamma_N)]^T$.

### 3.2.2 2D-ESPRIT based DOA Estimation

The invariance structure [11] present in the steering matrix $A(\theta, \varphi)$ can be exploited to perform a 2D angle estimation for the angle of arrivals $(\theta_p, \varphi_p)$; $p = 1, \ldots, N$ as in [8] where a rectangular array was used for the joint estimation of azimuth and elevation angles. We define diagonal matrices $\Phi = \text{diag}(\Phi_1, \ldots, \Phi_N)$ and $\gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_N)$ and selection matrices

$$
J_{01} = [I_{(N-1)} \ 0_{(N-1)}] \quad J_{01} = J_{01} \otimes I_P
$$

$$
J_{02} = [0_{(N-1)} \ 1_{(N-1)}] \quad J_{02} = J_{02} \otimes I_P
$$

$$
J_{\varphi 1} = [I_{(\varphi - 1)} \ 0_{(\varphi - 1)}] \quad J_{\varphi 1} = I_N \otimes J_{\varphi 1}
$$

$$
J_{\varphi 2} = [0_{(\varphi - 1)} \ I_{(\varphi - 1)}] \quad J_{\varphi 2} = I_N \otimes J_{\varphi 2}
$$

(17)

where $I_L$ is an $L \times L$ identity matrix and $0_L \in \mathbb{R}^L$ is an $L$-dimensional vector of zeros. Using the subarray selection matrices in (17), the following invariance equations can be formed from the steering matrix

$$
J_{02}A = J_{01}A\Phi \quad J_{\varphi 2}A = J_{\varphi 1}A\gamma
$$

(18)

Since the array steering matrix spans the signal subspace, the following equations can be obtained after eigenvalue decomposition of the spatio-temporal correlation matrix

$$
J_{02}E_S = J_{01}E_S\xi \quad J_{\varphi 2}E_S = J_{\varphi 1}E_S\Psi
$$

(19)

where $E_S$ are the signal subspace eigenvectors corresponding to the $N$ largest eigenvalues of the covariance matrix. $\xi$ and $\Psi$ can be estimated via a least square solution of (19)

$$
\xi = ((J_{02}E_S)H(J_{02}E_S))^{-1}(J_{02}E_S)H(J_{01}E_S)
$$

$$
\Psi = ((J_{\varphi 2}E_S)H(J_{\varphi 2}E_S))^{-1}(J_{\varphi 2}E_S)H(J_{\varphi 1}E_S)
$$

(20)

It was shown in [8] that the eigendecomposition of $\xi$ and $\Psi$ can be expressed as

$$
\xi \equiv T^{-1}\Phi T \quad \Psi \equiv T^{-1}\gamma T
$$

(21)

Clearly, the DOAs can be estimated from (21) with an additional pairing step which can be achieved using the Mean Eigenvalue Decomposition pairing technique [9]. By adding the equations in (21), we have

$$
\xi + \Psi = T^{-1}(\Phi + \gamma)T
$$

(22)

The invariance matrices are then obtained from (21) as

$$
\Phi = T\xi T^{-1} \quad \gamma = T\Psi T^{-1}
$$

(23)
The direction cosine and normalized frequency are then evaluated as
\[ \Phi = \arg(\xi) \quad \gamma = \arg(\Psi) \tag{24} \]

Denoting
\[ \mu = \frac{\Phi}{2\pi \Delta r} + j\frac{\gamma}{2\pi \Delta x} \tag{25} \]
we use the definitions of \( \Phi \) and \( \gamma \) in (25) to obtain estimates of the AOAs as
\[ \hat{\theta} = \arg(\mu) \quad \text{and} \quad \hat{\zeta} = \arcsin(\mu) \tag{26} \]

The receive array steering vector is estimated by substituting the estimates of AOAs into (6).

### 3.3 Transmit Spatial Signature Estimation

Once angles of arrival (AOA), Doppler frequencies and receive array steering vectors have been estimated, the transmit spatial signature estimation is a least square fit to the channel for each transmit antenna. We assume that the complex amplitudes of the scattering sources are the same for each transmit antenna and all receive antennas and use the channel between each transmit antenna and the first receive antenna for the estimation. Using (2) and (10), the set of equations for the \( i \)th transmit antenna are
\[
\begin{bmatrix}
h_{1i}(1) \\
h_{2i}(2) \\ \vdots \\
h_{N_{R}i}(K)
\end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ \Pi_1 & \cdots & \Pi_{\hat{\rho}} \\ \vdots & \ddots & \vdots \\ \Pi_{K-1} & \cdots & \Pi_{K-1} \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \vdots \\ \alpha_{\hat{\rho}i} \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2) \\ \vdots \\ n(K) \end{bmatrix} \tag{27}
\]

where \( \Pi_{\rho} = \exp(j\omega_{\rho} \delta t) \) and \( \alpha_{\rho i} \) is the complex amplitude for the \( \rho \)th ray. A matrix representation of (27) is
\[
z_{i} = Fa_{i} + n \tag{28}
\]

where \( z_{i} \) is a vector containing the \( K \) known samples of \( h_{ji} \), \( F \) is the vandermode structured matrix in (27) and \( n \) is the noise vector. The complex amplitudes are the least square solution of (27)
\[
\hat{a}_{i} = (F^H F + \eta I)^{-1} F^H z_{i} \tag{29}
\]

where \( \eta \) is a regularizing parameter\(^1\). We solve (29) for the \( N_{T} \) transmit antennas and form the transmit signature for \( \rho \)th ray as \( v_{\rho} = [\alpha_{\rho1}, \cdots, \alpha_{\rho N_{T}}] \).

### 3.4 Prediction

Once the parameters of the model have been estimated, estimation and prediction of the CSI is done by substituting the parameters into the model for the desired time instants
\[
\hat{H}(\tau) = \sum_{\rho=1}^{\hat{\rho}} b_{\rho}(\hat{\theta}_{\rho}) v_{\rho}^T \exp(j\hat{\omega}_{\rho} \tau) \tag{30}
\]

where \( \tau \) denotes the time instant for which the CSI is to be estimated or predicted.

\(^1\)This parameter is introduced to make the predictor robust by reducing sensitivity to the particular values of \( F \).
Table 1: Simulation Parameters for WINNER II SCM MIMO model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Drops (Realizations)</td>
<td>1000</td>
</tr>
<tr>
<td>Sampling Density</td>
<td>10 per $\lambda$</td>
</tr>
<tr>
<td>Mobile Speed</td>
<td>50 kmh$^{-1}$</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2.0 GHz</td>
</tr>
<tr>
<td>Scenario</td>
<td>Urban Macro (UMA)</td>
</tr>
<tr>
<td>Transmit Antenna Array</td>
<td>ULA</td>
</tr>
<tr>
<td>Receive Antenna Array</td>
<td>ULA</td>
</tr>
</tbody>
</table>

4 Numerical Simulations

In this section, we analyze the performance of the proposed algorithm and compare with the SISO-ESPRIT based schemes (MEHaM and MECoM) in [15] and the AR model with beamspace transformation scheme [13] which we call AR-Beam. The prediction error of the algorithms is evaluated using the normalized mean squared error (NMSE) criterion

$$\text{NMSE}(\tau) = \frac{\mathbb{E}[||\hat{H}(t+\tau) - H(t+\tau)||_F^2]}{\mathbb{E}[||H(t+\tau)||_F^2]}$$

$$\approx \frac{1}{M} \sum_{m=1}^{M} \sum_{k=1}^{K} \frac{||\hat{H}(t+\tau) - H(t+\tau)||_F^2}{||H(t+\tau)||_F^2}$$

(31)

where $M$ is the number of snapshots and $|| \cdot ||_F$ denotes the Frobenius norm. The predictions were performed using the WINNER II/3GPP SCM model [1] with parameters in Table 1. Figures 1 and 2 present the NMSE comparison of the proposed algorithm with the SISO-ESPRIT and AR beamspace schemes respectively. In Fig. 1, the proposed scheme results in an improvement over MEHaM and MECoM. Similarly, in Fig. 2, the proposed ESMIP scheme outperforms the AR-Beam approach at all noise levels as expected since it utilizes the spatial information. As expected, increasing the SNR improves the prediction accuracy. However for SNR above 10 dB, the performance improvement is only noticeable for the first few predictions. The CDF of the prediction NMSE of ESMIP for a prediction interval of 5ms ($\approx 0.5 \lambda$) is shown in Fig. 3. Fig. 4 shows the NMSE of the proposed predictor initialized with observation length between $1\lambda$ and $5\lambda$ at SNR of 5 dB. Clearly, increasing the observation length increases prediction accuracy. However, for observation lengths above $3\lambda$, the performance does not continue to improve significantly. Fig. 5 compares the performance of the proposed algorithm for the reconstruction of the channel over the estimation interval with the noisy channel estimates. We observe that the proposed predictor offer significance improvement in channel estimates at low SNR and can be used with common channel estimation schemes to improve the overall system performance.

5 Conclusion

In this paper, we proposed an ESPRIT based parametric prediction scheme that exploits both temporal and spatial correlations for realistic narrowband MIMO spatial channel models. The proposed predictor exploits all available temporal and spatial correlations by using a 2D-ESPRIT to overcome angle ambiguity and improve parameter estimation. It was shown that our algorithm offers improved performance in terms of NMSE and achievable prediction length over existing algorithms. Future work will include evaluation of prediction accuracy in terms of overall system performance.
Figure 1: The NMSE of ESMIP and SISO ESPRIT Schemes for $4 \times 4$ MIMO Channel Prediction versus prediction length at different SNR.

Figure 2: The NMSE of ESMIP and AR-Beamspace Approach [13] for $4 \times 4$ MIMO Channel Prediction versus prediction length at different SNR.

Figure 3: The CDF of prediction NMSE of ESMIP on the WINNER II model with parameters in Table 1 for a prediction length of 5 ms ($\approx 0.5\lambda$) at different SNR. Prediction initialized with observation length of $3\lambda$.

References


Figure 4: The prediction NMSE of ESMIP for different observation lengths at SNR=5 dB.

Figure 5: The NMSE of ESMIP and Channel Estimates versus Observation segment length in WINNER II model for UMA with \( v = 50 \text{ Km/h} \) at various SNR. C.EST denotes NMSE between noisy estimates and actual channel.


