TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS — 2008

END-OF-YEAR

COMP 202 / SWEN 202
Formal Methods of Computer Science / Formal Foundations of Software Engineering

Time Allowed: 3 Hours

Instructions:

- There are six (6) questions, each worth 30 marks.
- Answer all of the questions.
- The exam will be marked out of one hundred and eighty (180).
- Calculators ARE NOT ALLOWED.
- Non-electronic Foreign language dictionaries are allowed.
- No other reference material is allowed.

Question 1. Finite acceptors [30 marks]

(a) [6 marks] Draw a transition diagram for an NFA (Nondeterministic Finite Acceptor) that accepts the language defined by the regular expression $a^*(ab|acd)^*a^*$.

You are not required to use a systematic construction. Your NFA should be as simple as possible, and may include null transitions.

(b) [4 marks] Give a trace (sequence of transitions) showing the behaviour of your NFA when given the string *aabacdaaa* as input. Show all possible states that the NFA could be in at each step.

(c) [8 marks] Draw a transition diagram for a DFA (Deterministic Finite Acceptor) equivalent to your NFA from part **(a)**.

Explain how your DFA was constructed and how states of the DFA are related to those of the NFA.

(d) [2 marks] Give a trace (sequence of transitions) showing the behaviour of your DFA when given the string *abcda* as input.

(e) [10 marks] Prove that an NFA accepts a finite language if and only if its transition diagram, considered as a directed graph, contains no cycles consisting only of "useful" states. Recall that a state is "useful" if it lies on a path from the initial state to some accepting state. Explain why the result is false if the "useful" states condition is omitted.

Question 2. Describing languages [30 marks]

(a) [22 marks] Consider the following languages:

(i) The set of strings over $\{a, b, c\}$ containing exactly one a and exactly one b.

(ii) The set of strings of the form $a^m b^n$, where $m < n$.

(iii) The set of all palindromes over $\{a, b, c\}$; i.e. the set of all strings $\alpha \in \{a, b, c\}^*$ such that $\alpha^R = \alpha$, where α^R is the reflection (or reverse) of α.

(iv) The set of strings over $\{a, b, c, d\}$ such that no c occurs before an a and no b occurs after a d.

For each of the above languages: say whether the language is regular or not regular. If the language is regular, write a regular expression that defines it; otherwise, prove that it is not regular.

(b) [8 marks] The following grammars all define a simple language of arithmetic expressions with binary operators $+$ and $*$, and numbers 1, 2 and 3 as atomic expressions:

G1:
$$\begin{aligned} E &\to E + E \mid T \\ T &\to F * F \mid F \\ F &\to 1 \mid 2 \mid 3 \end{aligned}$$

G2:
$$\begin{aligned} E &\to T + E \mid T * E \mid T \\ T &\to 1 \mid 2 \mid 3 \end{aligned}$$

G3:
$$\begin{aligned} E &\to T + E \mid T \\ T &\to F * T \mid F \\ F &\to 1 \mid 2 \mid 3 \end{aligned}$$

G4:
$$\begin{aligned} E &\to T + E \mid E * T \mid T \\ T &\to 1 \mid 2 \mid 3 \end{aligned}$$

G5:
$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to 1 \mid 2 \mid 3 \end{aligned}$$

G6:
$$\begin{aligned} E &\to T + T \mid T * T \mid T \\ T &\to 1 \mid 2 \mid 3 \end{aligned}$$

G7:
$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to 1 \mid 2 \mid 3 \end{aligned}$$

G8:
$$\begin{aligned} E &\to E + T \mid E * T \mid T \\ T &\to 1 \mid 2 \mid 3 \end{aligned}$$

(i) [2 marks] Which grammar treats $+$ and $*$ as having the same precedence, and both being non-associative?

(ii) [2 marks] Which grammar treats $+$ and $*$ as having the same precedence, and both being left-associative?

(iii) [2 marks] Which grammar treats $+$ and $*$ as having different precedence, and both being right-associative?

(iv) [2 marks] Which grammar treats $+$ and $*$ as having different precedence and different-associativity?

Question 3. Parsing [30 marks]

Consider the following grammar:

$$
\begin{array}{lcl}
\textit{Stmt} & \rightarrow & \texttt{if}\ \textit{Expr}\ \texttt{then}\ \textit{Stmt}\ \texttt{else}\ \textit{Stmt}\ \texttt{fi} \\
& | & \texttt{if}\ \textit{Expr}\ \texttt{then}\ \textit{Stmt}\ \texttt{fi} \\
& | & \textit{ID}\ \texttt{:=}\ \textit{ID} \\
\\
\textit{Expr} & \rightarrow & \textit{ID}\ \texttt{==}\ \textit{ID} \\
\\
\textit{ID} & \rightarrow & \texttt{a} \mid \texttt{b} \mid \texttt{c} \mid \ldots
\end{array}
$$

(a) [5 marks] For each of the following strings, discuss whether the above grammar produces it or not. Give *parse trees* as evidence where possible.

(i) `if a == b then a := c ; a := b fi`

(ii) `if a == b then if b == c then a := c fi fi`

(b) [5 marks] Compute the following sets:

(i) *first(Expr)*

(ii) *follow(Stmt)*

(c) [5 marks] By computing *first()* sets appropriately, show that this grammar is not LL(1).

(d) [5 marks] By *left-factoring*, convert this grammar to be LL(1).

Now, consider the following grammar:

$$
\begin{array}{lcl}
\textit{Expr} & \rightarrow & \textit{Expr} + \textit{Term} \mid \textit{Term} \\
\\
\textit{Term} & \rightarrow & \textit{ID} \\
\\
\textit{ID} & \rightarrow & \texttt{a} \mid \texttt{b} \mid \texttt{c} \mid \ldots
\end{array}
$$

(e) [5 marks] The above grammar is *left-recursive*. Discuss why this means it is not LL(1).

(f) [5 marks] Show how the grammar can be rewritten to eliminate left-recursion. Indicate any differences in the parse trees produced, compared with the original grammar.

Question 4. FlowChart Programs [30 marks]

(a) Indicate whether each of the following FlowChart programs is *syntactically correct* or not. If it is not correct, indicate why; otherwise, briefly describe what it does.

(i) [3 marks]

```
    input x;
    input y;
    if x < y then 1 else 2;
1:  x := y;
    goto 3;
2:  skip;
```

(ii) [3 marks]

```
    input x;
    if x < 0 then 1 else 2;
1:  x := -x;
2:  skip;
    output x;
```

(iii) [3 marks]

```
    input x;
    input y;
    if x < 0 then 1 else 4;
1:  if y < 0 then 2 else 3;
2:  z := x + y;
3:  goto 5;
4:  z := 0;
5:  output z;
```

(b) Consider the following two FlowChart programs.

```
    x := 1;
    y := 0;
1:  if x < 3 then 2 else 3;
2:  x := x + 1;
    y := y + 1;
    output y;
    goto 2;
3:  skip;
```

(Program A)

```
    x := 1;
    y := 0;
1:  if x < 3 then 2 else 3;
2:  x := x + 1;
    y := x - 1;
    output y;
    goto 2;
3:  skip;
```

(Program B)

(i) [3 marks] Give an execution trace for program A.

(ii) [4 marks] Briefly discuss whether these programs are *weakly equivalent*, *strongly equivalent* or *not equivalent*.

(c) The FlowChart language is to be extended with support for *arrays*. This allows programs such as the following, where an element of the array a is overwritten with zero:

$$
\begin{array}{l}
a := \langle 0, 1, 2, 3 \rangle; \\
\textbf{input } x; \\
a[x] := 0;
\end{array}
$$

(i) [2 marks] Give a condition which ensures any statement of the form "$v[i] := e$" will not cause the program to abort.

(d) The semantics for assignments of the form "$v := e$" is given by:

$$\frac{P(pc) = (l_1 : v := e) \quad n = \mathcal{V}(e, S)}{(P, pc, S) \longrightarrow (P, pc{+}1, S[v := n])} \quad \text{[ASSIGN]}$$

(i) [2 marks] On the following FlowChart Machine state, apply the above ASSIGN rule to produce the next state:

$$(\langle 1 : \ x \ := \ y \rangle, \ 1, \ \{x \mapsto 2, y \mapsto 3\})$$

(ii) [4 marks] Give a similar rule which governs assignments of the form "$v[i] := e$". Note, if v is an array, then $v[i := n]$ returns v with element i updated to hold the constant n.

(e) [6 marks] For each of the numbered statements in the following program, write an appropriate *assertion* which always holds true before that statement is executed. Indicate whether the postcondition will be satisfied after the return statement is executed.

```
PRE:
POST: ret ≥ 0
absum(list) begin
    i := 1;
    r := 0;
1:  if i≤|list| then 2 else 6;
2:  if list[i] < 0 then 3 else 4;
3:  r := r − list[i];
    goto 5;
4:  r := r + list[i];
5:  i := i + 1;
    goto 1;
6:  return r;
end
```

Question 5. Alloy [30 marks]

The statement "One Man's Ceiling is Another Man's Floor", taken from a song by Paul Simon, inspired the following Alloy model:

```
sig Platform {}
sig Man {ceiling, floor: Platform}

fact above { all m: Man | some n: Man | m.ceiling = n.floor }

pred below { all m: Man | some n: Man | m.floor = n.ceiling }
pred woleb { some n: Man | all m: Man | m.floor = n.ceiling }
```

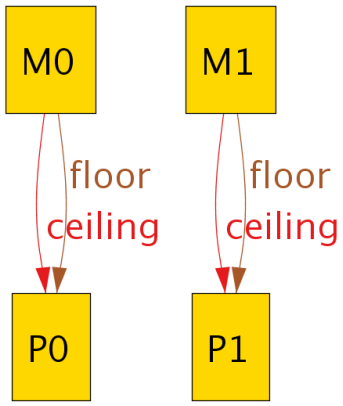
(a) Consider the following instance of the model:

$$\texttt{Man} = \{M0, M1\}$$

$$\texttt{Platform} = \{P0, P1\}$$

$$\texttt{ceiling} = \texttt{floor} = M0\texttt{->}P0 + M1\texttt{->}P1 = \begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline \end{array}$$

(i) [2 marks] Draw a visualisation (graph representation as Alloy would do) of this model instance.



(ii) [1 mark] Compute $M0.\texttt{ceiling}$.

$$M0.\begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline \end{array} = P0$$

(iii) [1 mark] Compute $M1.\texttt{floor}$.

$$M1.\begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline \end{array} = P1$$

(iv) [2 marks] Is the predicate called `below` true for this instance? Give a brief explanation of why or why not.

Predicate `below` is true for this instance since floor and ceiling are equal for all men.

(v) [2 marks] Is the predicate called `woleb` true for this instance? Give a brief explanation of why or why not.

Predicate `woleb` is not true for this instance since there is no man so that all men's floors are equal to his ceiling.

(b) Consider the following instance of the model:

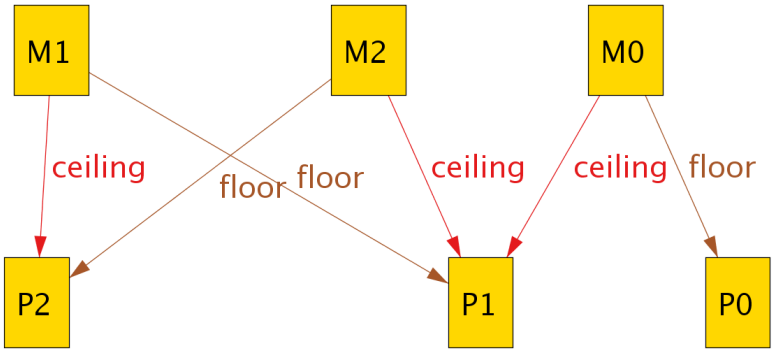
$$\texttt{Man} = \{M0, M1, M2\}$$

$$\texttt{Platform} = \{P0, P1, P2\}$$

$$\texttt{ceiling} = M0\texttt{->}P1 + M1\texttt{->}P2 + M2\texttt{->}P1 = \begin{array}{|c|c|}\hline M0 & P1 \\ \hline M1 & P2 \\ \hline M2 & P1 \\ \hline \end{array}$$

$$\texttt{floor} = M0\texttt{->}P0 + M1\texttt{->}P1 + M2\texttt{->}P2 = \begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline M2 & P2 \\ \hline \end{array}$$

(i) [2 marks] Draw a visualisation (graph representation as Alloy would do) of this model instance.



(ii) [2 marks] Compute `floor.~ceiling`.

$$\begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline M2 & P2 \\ \hline \end{array}\,.\,\sim\begin{array}{|c|c|}\hline M0 & P1 \\ \hline M1 & P2 \\ \hline M2 & P1 \\ \hline \end{array} = \begin{array}{|c|c|}\hline M0 & P0 \\ \hline M1 & P1 \\ \hline M2 & P2 \\ \hline \end{array}\,.\,\begin{array}{|c|c|}\hline P1 & M0 \\ \hline P2 & M1 \\ \hline P1 & M2 \\ \hline \end{array} = \begin{array}{|c|c|}\hline M1 & M0 \\ \hline M1 & M2 \\ \hline M2 & M1 \\ \hline \end{array}$$

(iii) [2 marks] Compute `^(floor.~ceiling)`.

$$\wedge \begin{array}{|c|c|}\hline M1 & M0 \\ \hline M1 & M2 \\ \hline M2 & M1 \\ \hline \end{array} = \begin{array}{|c|c|}\hline M1 & M0 \\ \hline M1 & M2 \\ \hline M2 & M1 \\ \hline M1 & M1 \\ \hline M2 & M0 \\ \hline M2 & M2 \\ \hline \end{array}$$

(iv) [2 marks] Compute `floor.~ceiling.Man`.

$$\begin{array}{|c|c|}\hline M1 & M0 \\ \hline M1 & M2 \\ \hline M2 & M1 \\ \hline \end{array} . \begin{array}{|c|}\hline M0 \\ \hline M1 \\ \hline M2 \\ \hline \end{array} = \begin{array}{|c|}\hline M1 \\ \hline M2 \\ \hline \end{array}$$

(v) [2 marks] In your own words, describe what `floor.~ceiling.Man` represents.

The set of tuples of Men so that the first one is above the second one, that is, the second man's ceiling is the first man's floor.

(vi) [2 marks] Is the predicate called `below` true for this instance? Give a brief explanation of why or why not.

Predicate `below` is not true for this instance since *M*0's floor is not the ceiling for a man.

(c) Extending the Alloy specification

(i) [2 marks] Add a fact to the Alloy model given above that makes sure that ceiling and floor are distinct for each Man.

```
fact  { no floor & ceiling }
```

(ii) [3 marks] Write an Alloy function called `fun_above` (with no arguments) that returns a binary relation between a platform and any platform directly above it. A platform `q` is directly above a platform `p`, i.e. `p->q in fun_above`, if and only if there is a Man who has Platform `p` as floor and Platform `q` as ceiling.

```
fun foo: Platform -> Platform {
  ~floor.ceiling
}
```

(iii) [3 marks] Write a predicate called `notAboveItself` that is true if no Platform is (directly or indirectly) above itself. In other words, predicate `notAboveItself` should be true if and only if no platform is directly above itself or is directly above a Platform that is directly above itself, etc.

```
pred notAboveItself {
  no p: Platform | p in p.^fun_above
}
```

(iv) [2 marks] What would happen if you executed the command `run notAboveItself` using the Alloy analyser? Give a brief explanation of your answer.

No instances involving Men would be found since all instances satisfying predicate `notAboveItself` and fact `above` consist of an infinite number of Man.

Question 6. An Alloy Model for Finite Acceptors [30 marks]

A *Nondeterministic Finite Acceptor* (NFA) is defined as a tuple (Q, q, A, N, F), where Q is a non-empty finite set of states, $q \in Q$ is the initial state, A is a finite set of symbols, $N \subseteq Q \times A \times Q$ is the transition relation, and $F \subseteq Q$ is the set of final states.

Consider the following incomplete Alloy model for NFAs:

```
sig Symbol {}
sig State { transitions: Symbol -> State }
```

(a) [5 marks] Extend the Alloy model given above with a representation for start and final states according to the NFA definition given. Make sure that each NFA model instance has exactly one initial state.

```
one sig Init in State {}
sig Final in State {}
```

(b) Provide definitions (predicates) for the following classes of NFAs:

(i) [2 marks] NFAs with only one state.

```
pred oneState { one State }
```

(ii) [2 marks] NFAs with no final states.

```
pred noFinal { no Final }
```

(iii) [2 marks] NFAs for which the initial state is not final.

```
pred initNotFinal { no Init & Final }
```

(iv) [2 marks] NFAs without *self-loops*, i.e. transitions that start and end at the same state.

```
pred noSelfLoops { all s: State | no s & s.transitions[Symbol] }
```

(v) [2 marks] *Deterministic Finite Acceptors*, that is NFAs that do not have multiple transitions that start from the same state and are labelled with the same symbol.

```
pred det { all s: State, a: Symbol | lone s.transitions[a] }
```

(vi) [2 marks] NFAs that have a transition to a final state from each non-final state.

```
pred transToFinal { all s: State-Final | some s.transitions[Symbol] & Final }
```

(vii) [3 marks] NFAs for which all states are *reachable*, that is, can be reached from the initial state by following transitions only. More formally: Given a NFA (Q, q, A, N, F), a state $s \in Q$ is reachable if there is a sequence of states $q_0, \ldots, q_n$ such that $q_0 = q$, $q_n = s$, and for each $0 \leq i < n$ there is a symbol $a \in A$ such $(q_i, a, q_{i+1}) \in N$.

```
pred allReachable { State in Init.*{x,y: State | y in x.transitions[Symbol] } }
```

(c) Next you want to model relations and operations on NFAs but the current model is static and does not allow such dynamic behaviour.

(i) [5 marks] Rewrite the static model given above to provide a dynamic model that supports multiple NFAs. To support operations on NFAs, your new model should allow NFAs to have states and symbols in common, but with different start and final states as well as different transition relations.

```
sig Symbol, State {}
sig NFA {
  Q: set State,
  q: Q,
  A: set Symbol,
  N: Q->A->Q,
  F: set Q
}
```

(ii) [2 marks] Provide a predicate that checks whether two NFAs have the same set of states and the same set of symbols but different initial states.

```
pred pii(x, y: NFA) {
  x.Q = y.Q
  x.A = y.A
  x.q != y.q
}
```

(iii) [3 marks] Provide an Alloy operation that, given a NFA, a state q, and a symbol a, adds to the given NFA a transition labelled a from the initial state to state q. More formally: Given an NFA (Q, q, A, N, F), a state $s \in Q$ and a symbol $a \in A$, your operation should construct the NFA: $(Q, q, A, N \cup \{(q, a, s)\}, F)$.

```
pred piii(nfa, nfa': NFA, s: nfa.Q, a: nfa.A) {
  nfa'.Q = nfa.Q
  nfa'.q = nfa.q
  nfa'.A = nfa.A
  nfa'.N = nfa.N + nfa.q->a->s
  nfa'.F = nfa.F
}
```

****