# XMUT 202
# Digital Electronics

A/Prof. Pawel Dmochowski

School of Engineering and Computer Science
Victoria University of Wellington

# Week 9 Lectures 3 and 4

- Logic Gates (cont'd)
- In Lectures 1 and 2 we have learned about:
  - AND, OR, NOT Boolean operations
  - Logic circuit diagrams
  - Truth tables

- Today (lectures 3 and 4)
  - NOR and NAND gates
  - Boolean Theorems (for simplifying Boolean expressions)
  - De Morgan's Theorems (for simplifying Boolean expressions)

# Implementing Circuits From Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

# Implementing Circuits from Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

- Simple expression: $x = A \cdot B \cdot C$

# Implementing Circuits from Boolean Expressions

- It is important to be able to draw a logic circuit from a Boolean expression.

- Simple expression: $x = A \cdot B \cdot C$

- A more complex example expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

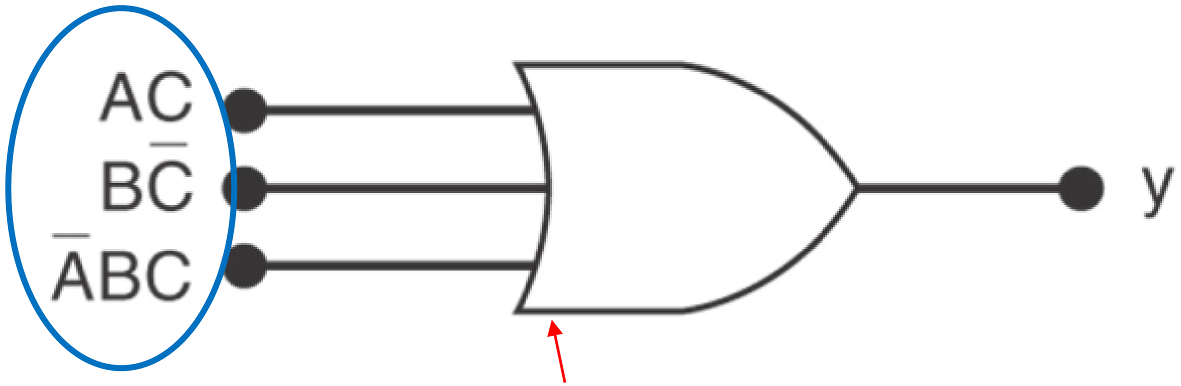
# Logic circuit for the Boolean expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

# Logic circuit for the Boolean expression:

$$y = AC + B\overline{C} + \overline{A}BC$$

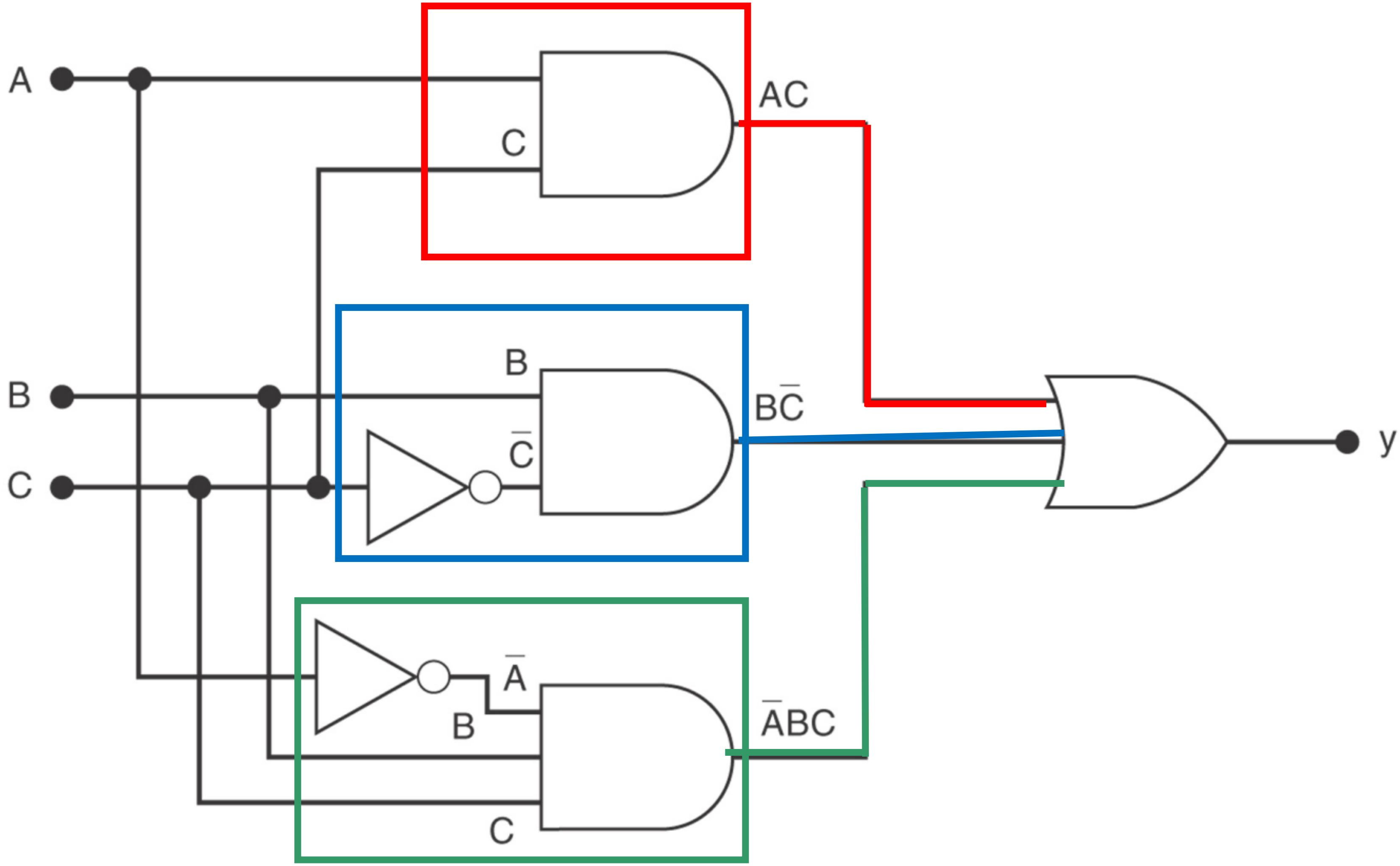
OR operations




3 inputs

OR gate

# Logic circuit for the Boolean expression:




$$y = AC + B\overline{C} + \overline{A}BC$$

# NOR Gate

- Combine basic AND, OR, and NOT operations.

# NOR Gate

- Combine basic AND, OR, and NOT operations.

- The NOR gate is an inverted OR gate. An inversion “bubble” is placed at the output of the OR gate.

# NOR Gate

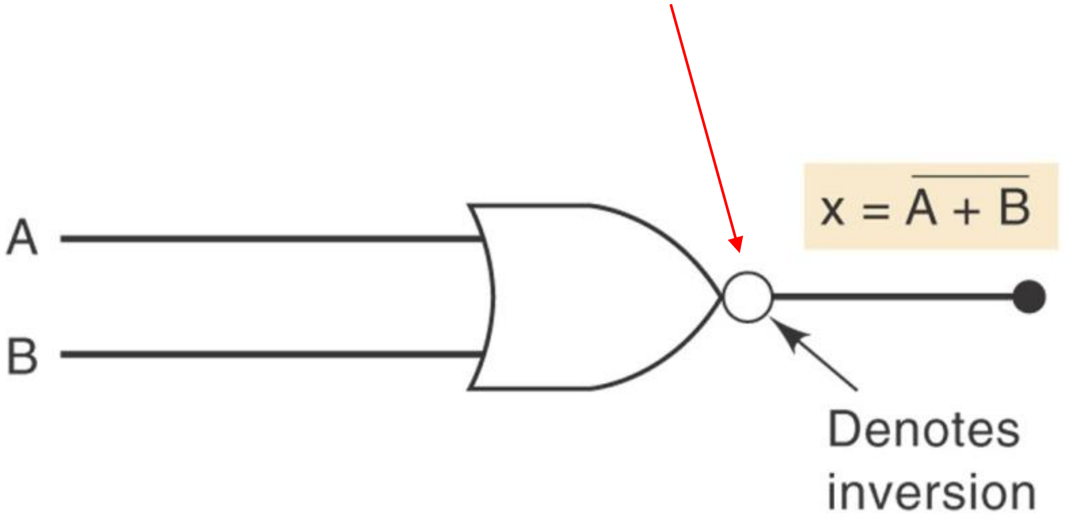
- Combine basic AND, OR, and NOT operations.

- The NOR gate is an inverted OR gate. An inversion “bubble” is placed at the output of the OR gate.

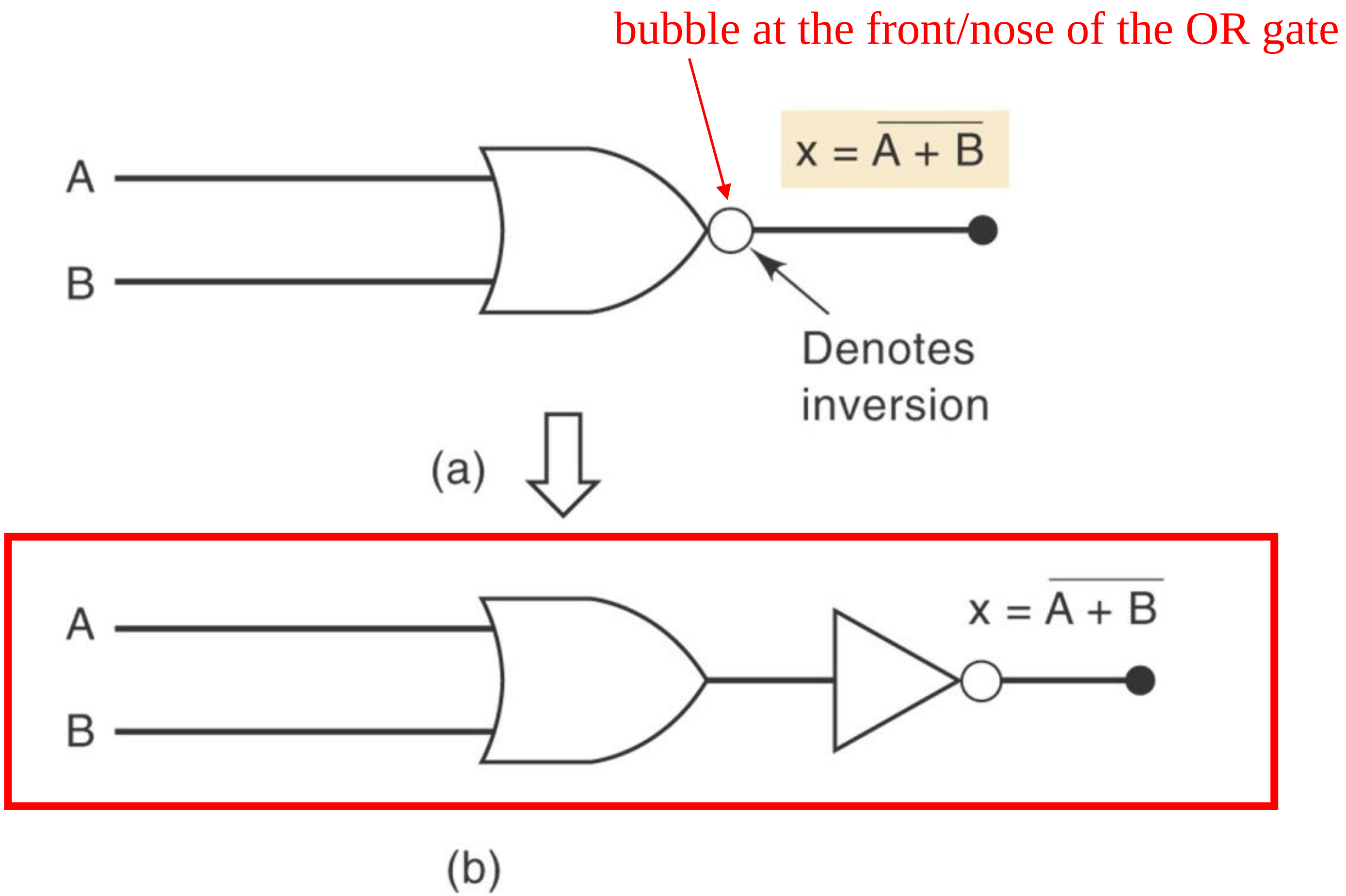
- The Boolean expression is $x = \overline{A + B}$

# (a) NOR symbol

Small circle at the front/nose of the OR gate

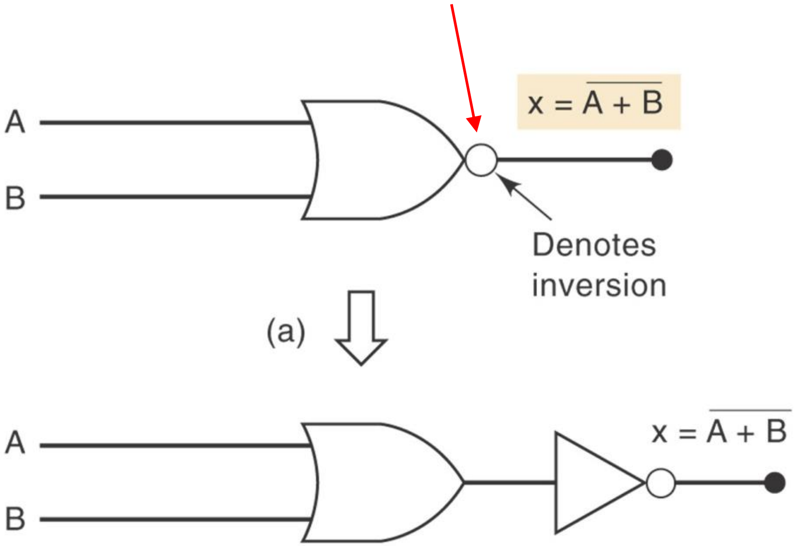


# (a) NOR symbol; (b) equivalent circuit



# (a) NOR symbol; (b) equivalent circuit; (c) truth table.

bubble at the front/nose of the OR gate




(a) (b)

| A | B | OR: $A + B$ | NOR: $\overline{A + B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

(c)

Truth table

# NAND Gate

- The NAND gate is an inverted AND gate.

# NAND Gate

- The NAND gate is an inverted AND gate.

- An inversion “bubble” is placed at the output of an AND gate.

# NAND Gate

- The NAND gate is an inverted AND gate.
- An inversion "bubble" is placed at the output of an AND gate.
- The Boolean expression is $X = \overline{AB}$

# NOR Gates and NAND Gates

- The output of NAND and NOR gates may be found by simply determining the output of an AND or OR gate and inverting it.

# NOR Gates and NAND Gates

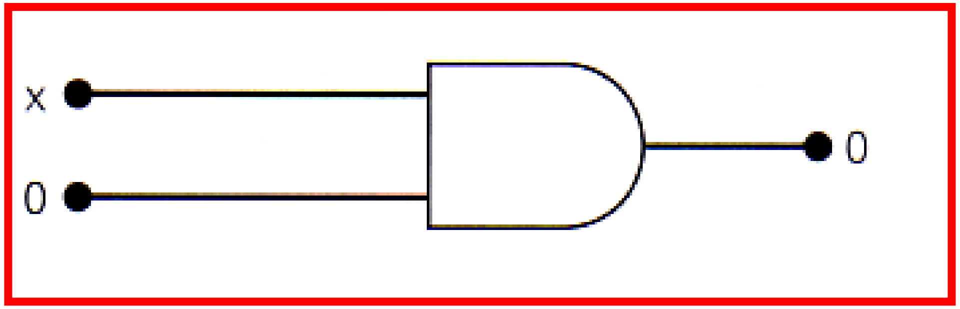
- The output of NAND and NOR gates may be found by simply determining the output of an AND or OR gate and inverting it.

- The truth tables for NOR and NAND gates show the complement of truth tables for OR and AND gates.

# Boolean Theorems

Single variable theorems

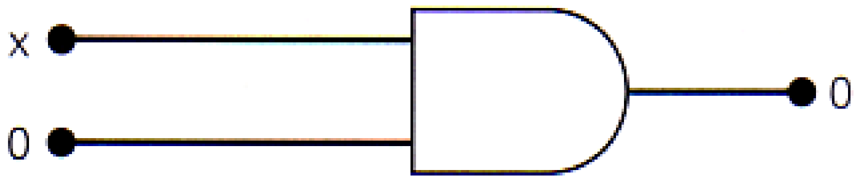
1) $x \cdot 0 = 0$



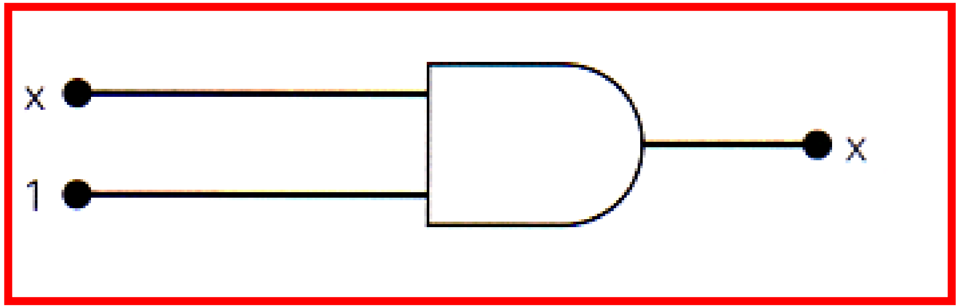
# Boolean Theorems

## Single variable theorems

1) $x \cdot 0 = 0$



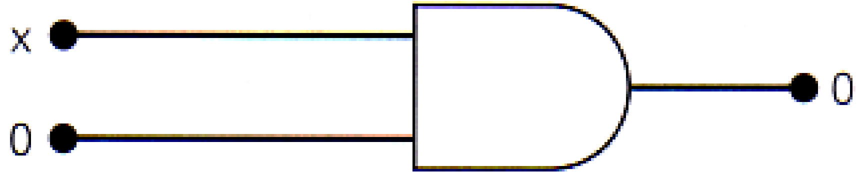

2) $x \cdot 1 = x$



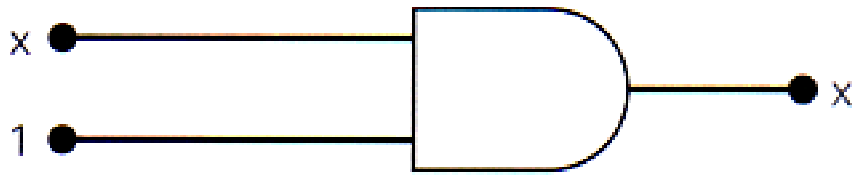
# Boolean Theorems

## Single variable theorems

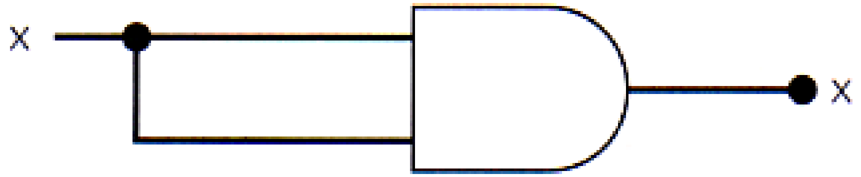
1) $x \cdot 0 = 0$



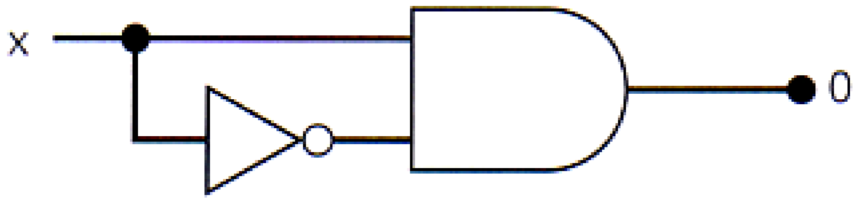

2) $x \cdot 1 = x$




3) $x \cdot x = x$



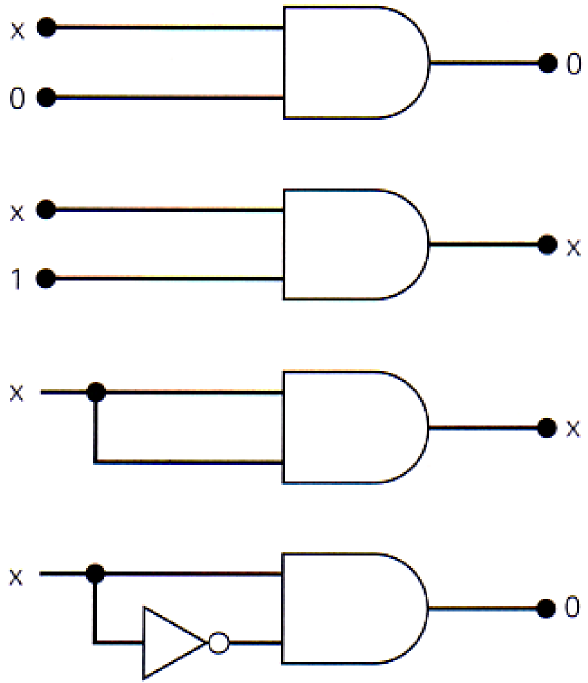

4) $x \cdot \overline{x} = 0$



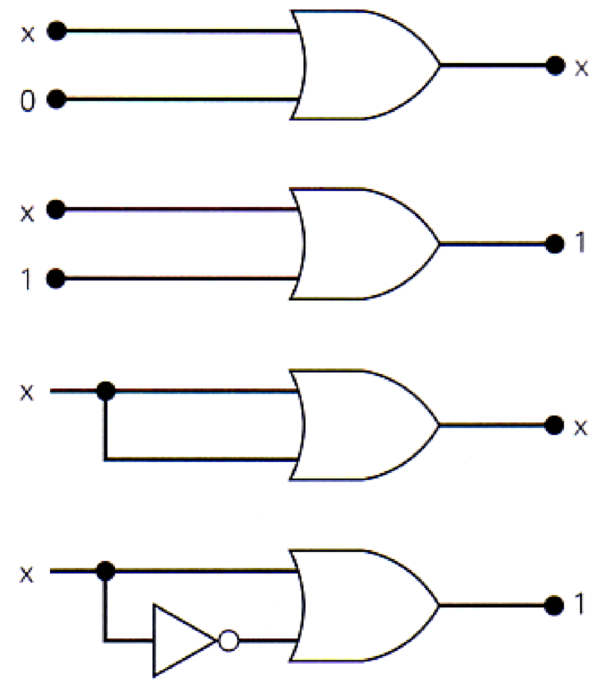
# Boolean Theorems

## Single variable theorems

1) $x \cdot 0 = 0$
2) $x \cdot 1 = x$
3) $x \cdot x = x$
4) $x \cdot \overline{x} = 0$




5) $x + 0 = x$
6) $x + 1 = 1$
7) $x + x = x$
8) $x + \overline{x} = 1$



# Boolean Theorems

Multivariable theorems:

1) $x + y = y + x$

2) $x \cdot y = y \cdot x$

3) $x + (y + z) = (x + y) + z = x + y + z$

4) $x(yz) = (xy)z = xyz$

5) $x(y + z) = xy + xz$

6) $(w + x)(y + z) = wy + xy + wz + xz$

7) $x + xy = x$

8) $x + \overline{x}y = x + y$

9) $\overline{x} + xy = \overline{x} + y$

# De Morgan’s Theorems

- When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them.

# De Morgan’s Theorems

- When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them.

- When the AND product of two variables is inverted, it is equivalent to inverting each variable individually and ORing them.

# De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \overline{x}.\overline{y}$$

# De Morgan's Theorems

- A NOR gate is equivalent to an AND gate with inverted inputs.

$$\overline{(x + y)} = \overline{x}.\overline{y}$$

- A NAND gate is equivalent to an OR gate with inverted inputs.

$$\overline{(x.y)} = \overline{x} + \overline{y}$$

# Universality of NAND and NOR gates

- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and NOT also known as the INVERTER)

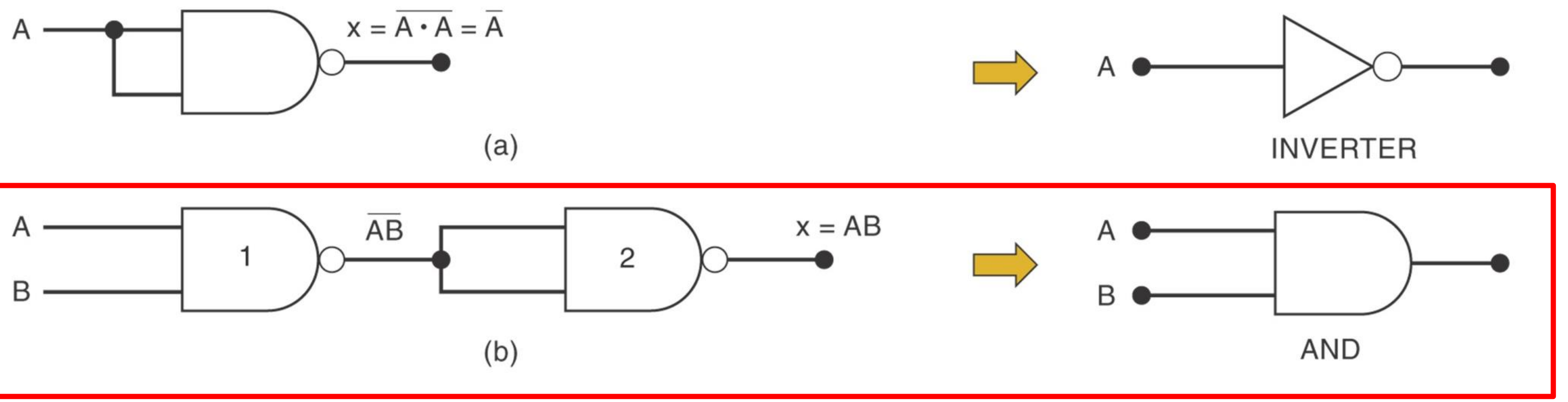
# Universality of NAND and NOR gates

- NAND or NOR gates can be used to create the three basic logic operations (OR, AND, and INVERTER)

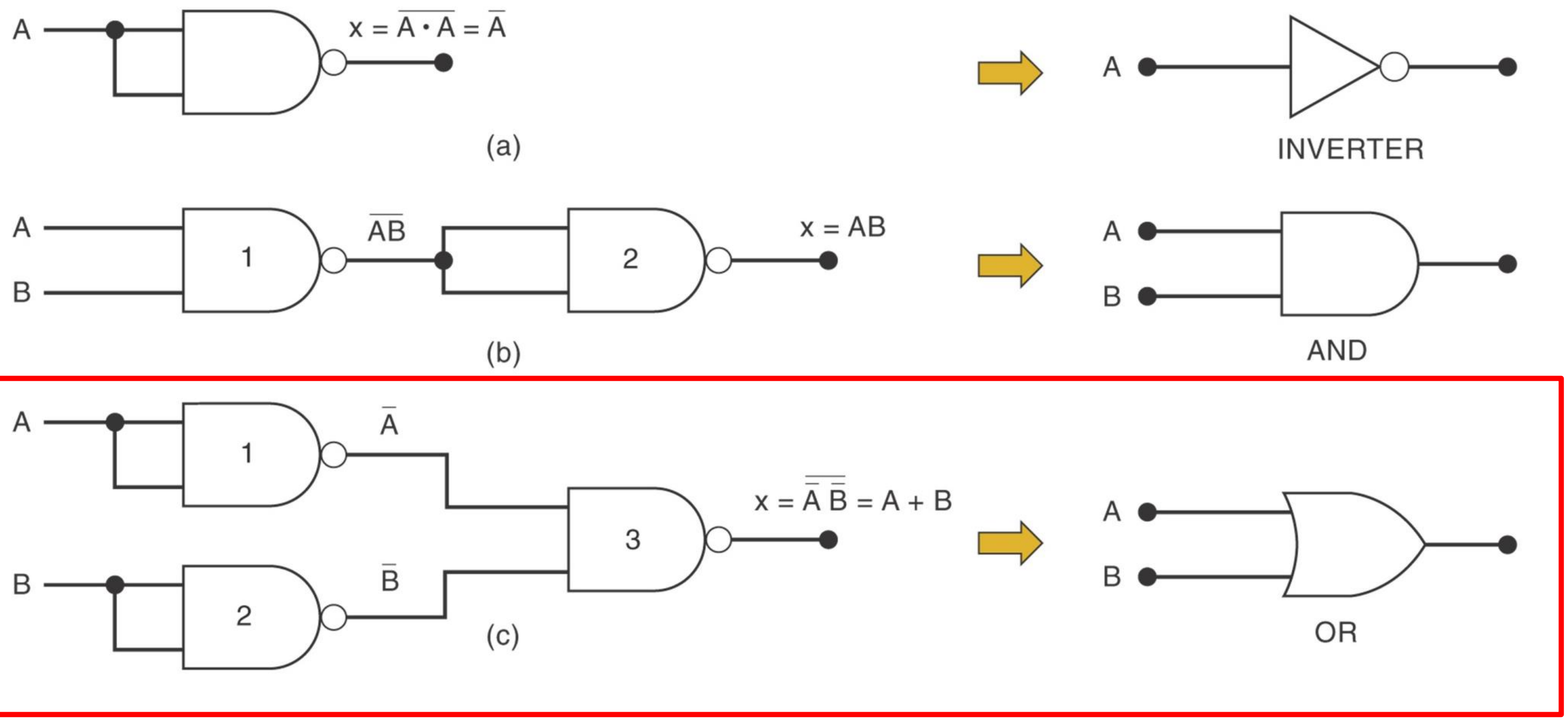
- How combinations of NANDs or NORs are used to create the three basic logic operations.

# NAND gates can be used to implement any Boolean function.

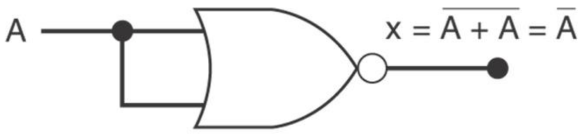
# NAND gates can be used to implement any Boolean function.



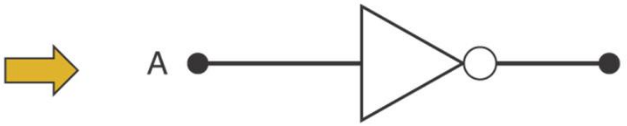
# NAND gates can be used to implement any Boolean function.



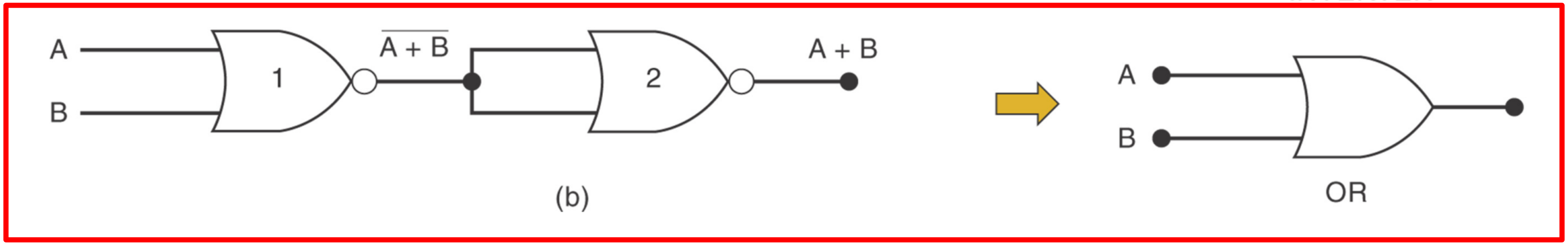
# NOR gates can be used to implement any Boolean function.




(a)



# NOR gates can be used to implement any Boolean function.

## NOR gates can be used to implement any Boolean function.