XMUT 101
Engineering Technology

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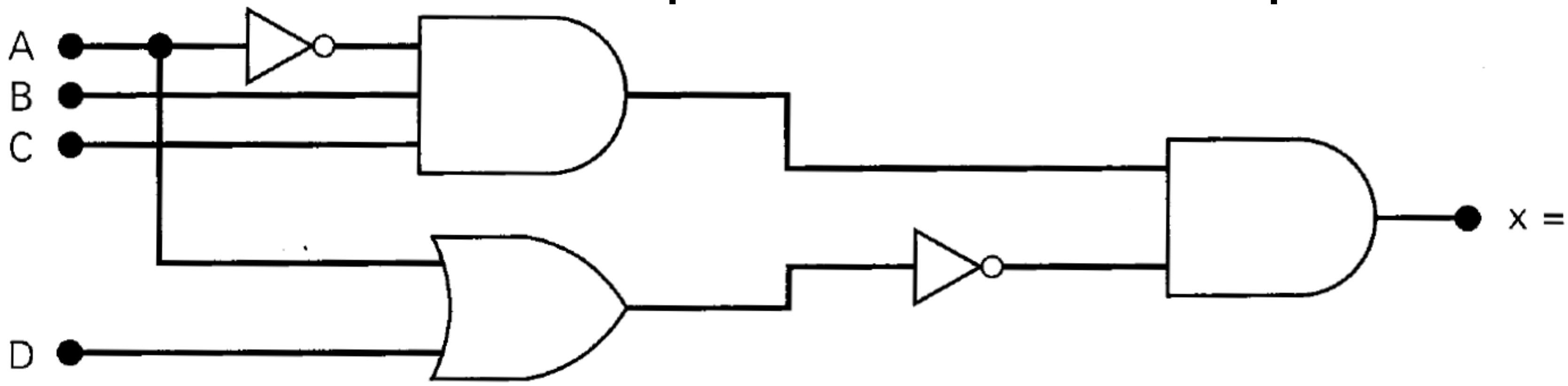
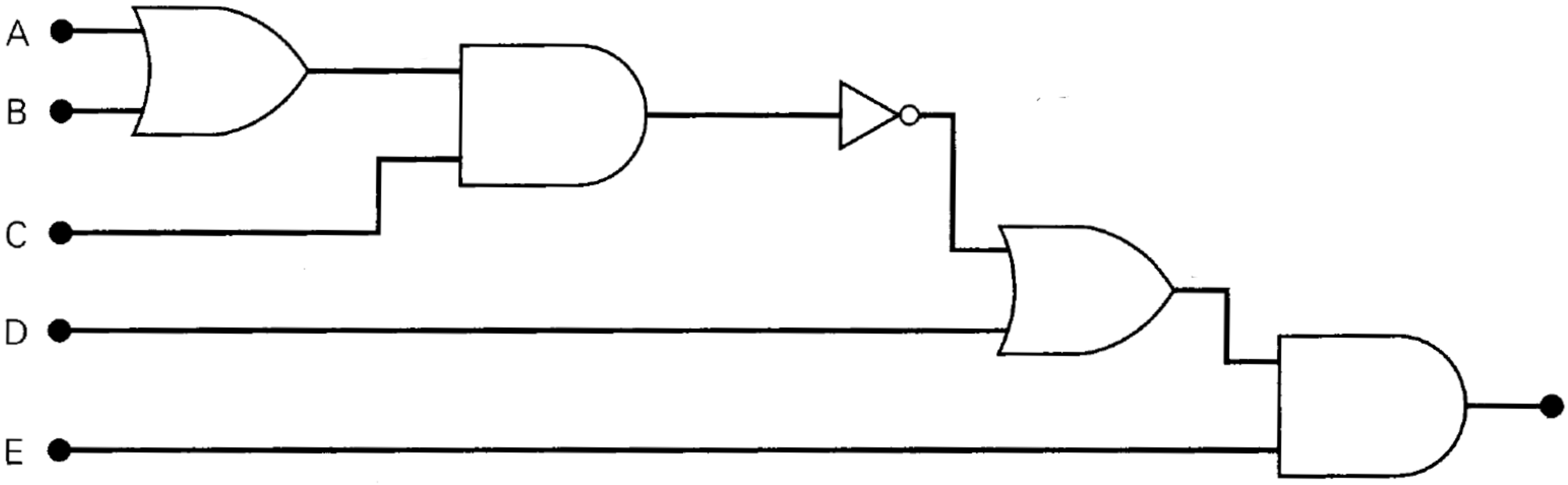
Victoria
UNIVERSITY OF WELLINGTON
Te Whare Wānanga
o te Ūpoko o te Ika a Māui
CAPITAL CITY UNIVERSITY

Review

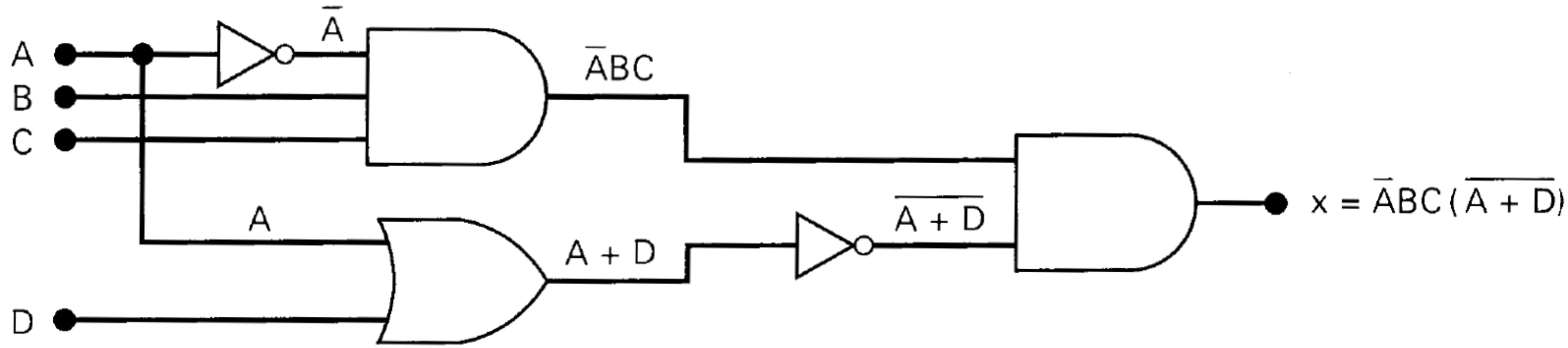
Write the Boolean expression for the output X

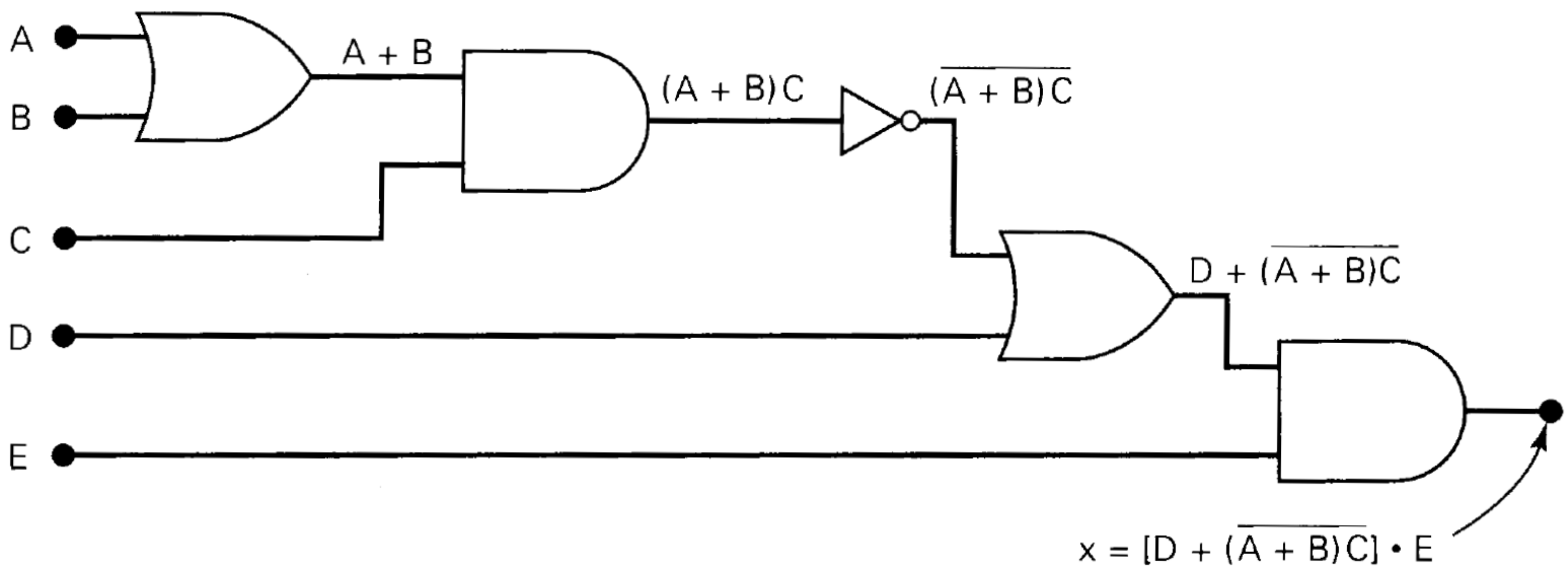




Review

Evaluate a) for A=0, B=1, C=1, D=1 and b) A=0, B=0, C=1, D=1, E=1





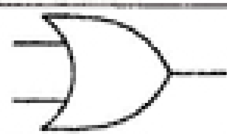
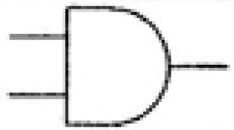
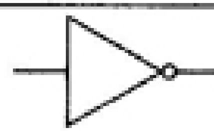
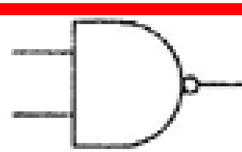
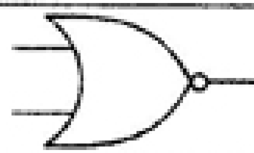
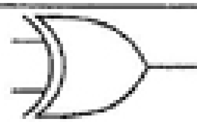
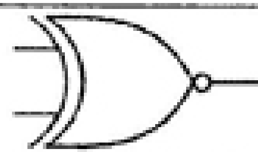
Review

- Draw a circuit diagram to implement $x=(A+B)(\overline{B}+C)$
- Draw a circuit diagram to implement $x=AC+B\overline{C}+\overline{A}BC$

Review

- Draw a circuit diagram to implement $x=\overline{AB(\overline{C+D})}$ and determine the output for A=B=C=1 and D=0

Logic Gates Symbols

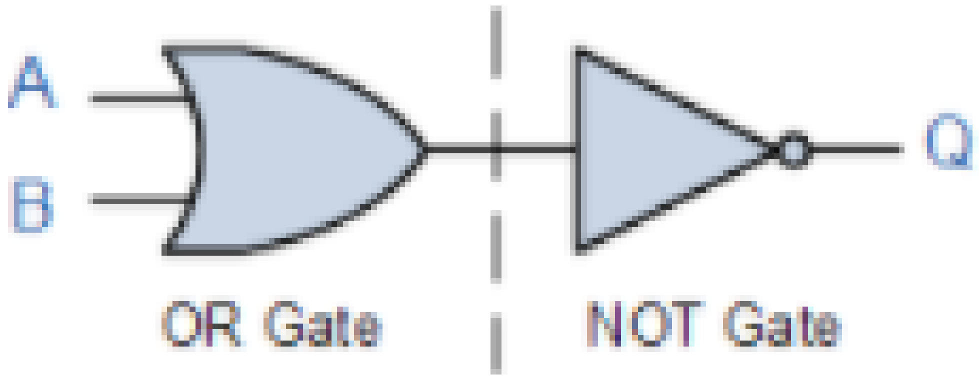
Gate	Symbol
OR	
AND	
NOT	
NAND	
NOR	
EX-OR or X-OR	
EX-NOR or X-NOR	

Exclusive OR

Exclusive NOR

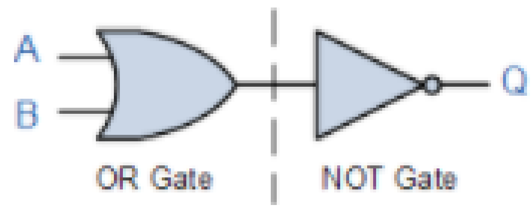
NOR Gates

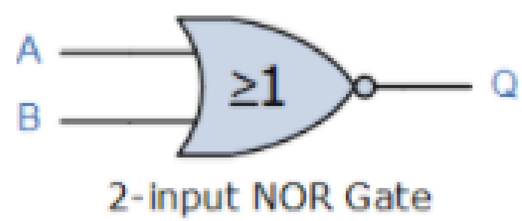
- Combine basic OR and NOT gate.
- Not OR - NOR gate is an inverted OR gate.



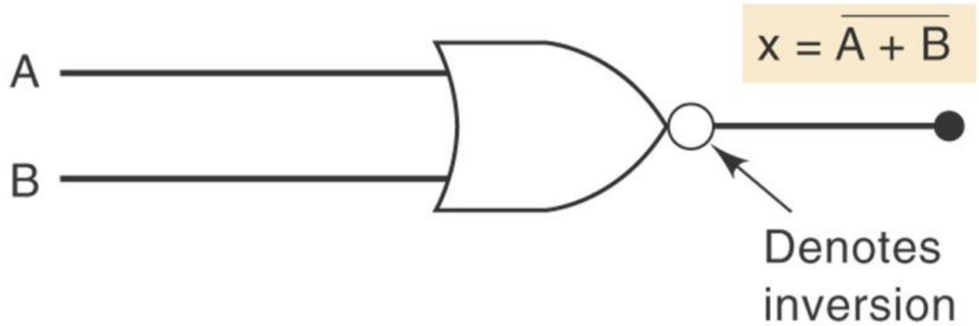
NOR Gates

- Combine basic OR and NOT gate.
- Not OR - NOR gate is an inverted OR gate.
- An inversion “bubble” is placed at the output of the OR gate.

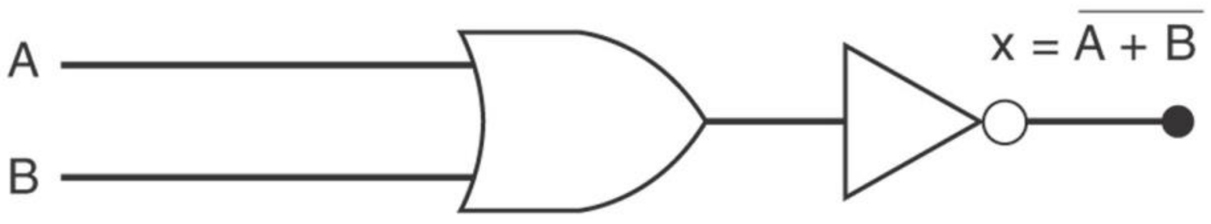




(a) NOR symbol



(b) Equivalent circuit; (c) Truth table.



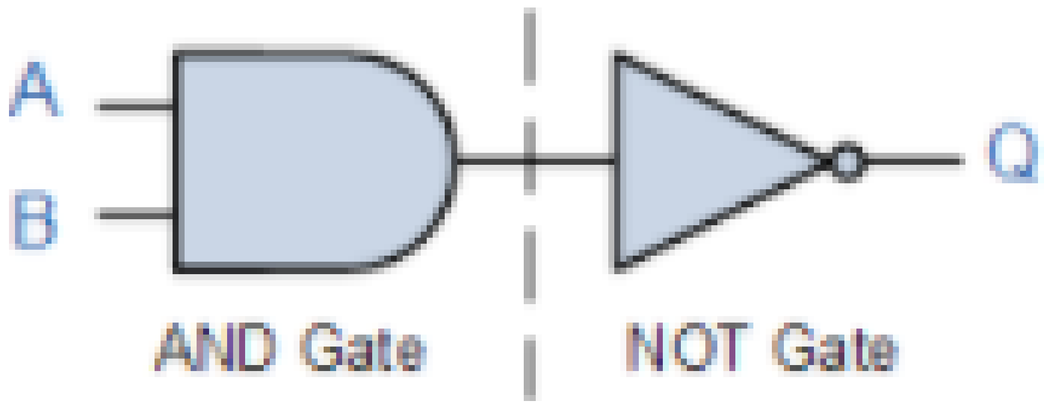
(b)

		OR	NOR
A	B	A + B	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

(c)

NAND Gates

- The NAND gate is an inverted AND gate.



NAND Gates

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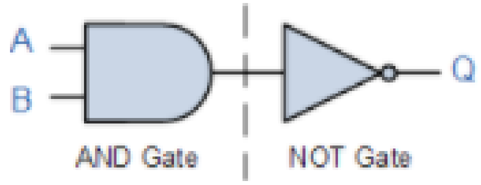
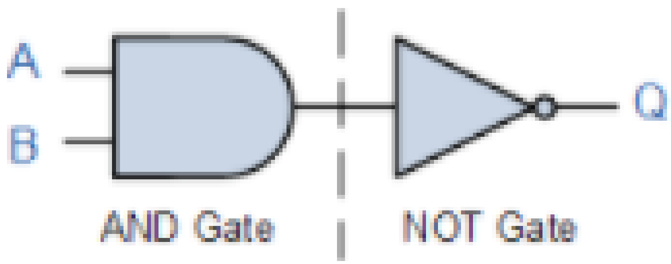


2-Input NAND Gate

NAND Gates



2-Input NAND Gate

NAND Truth table

A	B	$A \bullet B$	$\overline{A \bullet B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Review

- Draw a circuit diagram to implement $x=\overline{AB(\overline{C+D})}$ using only NOR / NAND gates. Evaluate for A=B=C=1 and D=0

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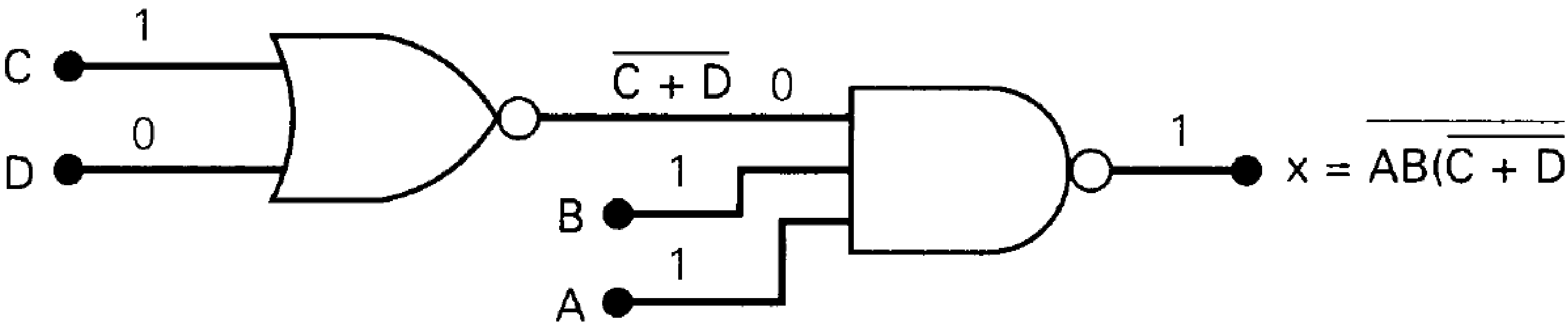
- Draw a circuit diagram to implement $x=\overline{AB(\overline{C+D})}$ using only NOR / NAND gates. Evaluate for A=B=C=1 and D=0



Boolean Algebra

- A *Boolean algebra* is defined as a closed algebraic system containing a set K of two or more elements and the two operators, . and +.
- Useful for identifying and *minimizing* circuit functionality
- Identity elements
  - a + 0 = a
  - a . 1 = a
- 0 is the identity element for the + operation.

  1 is the identity element for the . operation.

Boolean Algebra for OR gate

OR

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

Boolean Algebra for OR, AND

OR	***AND***
$0 + 0 = 0$	$0 \cdot 0 = 0$
$0 + 1 = 1$	$0 \cdot 1 = 0$
$1 + 0 = 1$	$1 \cdot 0 = 0$
$1 + 1 = 1$	$1 \cdot 1 = 1$

Boolean Algebra for OR, AND & NOT

OR	*AND*	*NOT*
$0 + 0 = 0$	$0 \cdot 0 = 0$	$\overline{0} = 1$
$0 + 1 = 1$	$0 \cdot 1 = 0$	$\overline{1} = 0$
$1 + 0 = 1$	$1 \cdot 0 = 0$	
$1 + 1 = 1$	$1 \cdot 1 = 1$	

Ordering Boolean Functions

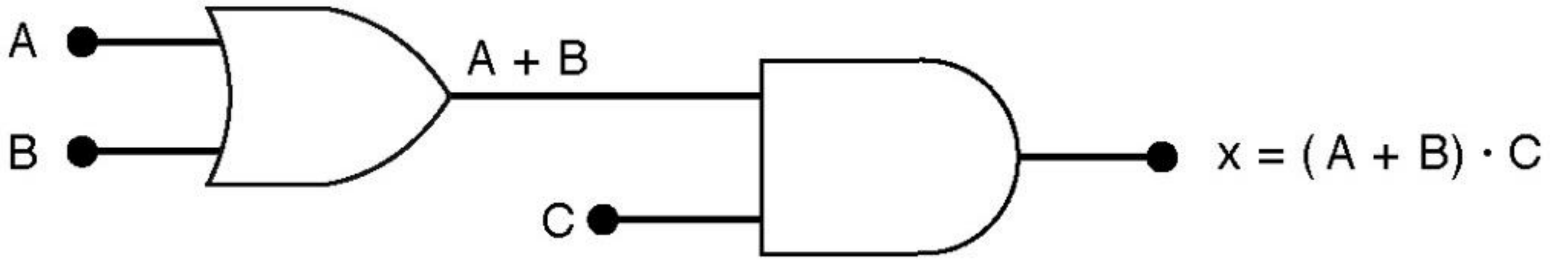
How to interpret A • B + C?

The above expression is read as:

A "AND" B "OR" C

Ordering Boolean Functions

- How to interpret A•B+C?
  - Is it A•B ORed with C ?
  - Is it A ANDed with B+C ?
- Order of precedence for Boolean algebra: AND before OR.
- Note that parentheses are needed here :



Commutativity and Associativity

- The Commutative Property:

  For every a and b,

  (1) $a + b = b + a$

  (2) $a \,.\, b = b \,.\, a$

- The Associative Property:

  For every a, b, and c,

  (1) $a + (b + c) = (a + b) + c$

  (2) $a \,.\, (b \,.\, c) = (a \,.\, b) \,.\, c$

Distributivity of the Operators and Complements

- The Distributive Property:

  For every a, b, and c in K,

  $a + (b . c) = (a + b) . (a + c)$

  $a . (b + c) = (a . b) + (a . c)$

- The Existence of the Complement:

  For every a in K there exists a unique element called $\bar{a}$

  (*complement of a*) such that,

  $$a + \bar{a} = 1$$

  $$a . \bar{a} = 0$$

Distributivity of the Operators and Complements

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- The Existence of the Complement:

  For every a in K there exists a unique element called a' (*complement of a*) such that,

  $a + \overline{a} = 1$

  $a . \overline{a} = 0$

- To simplify notation, the . operator is frequently omitted. When two elements are written next to each other, the AND (.) operator is implied…

$$a + b \bullet c = (a + b) \bullet (a + c)$$

$$a + bc = (a + b)(a + c)$$

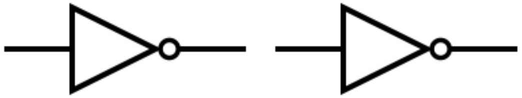
Duality

- The principle of *duality* is often a helpful concept – it is symmetry present properties and theorems.
- In Boolean algebra we form a dual expression
  - swap + for . operators, and vice versa
  - swap 1s for 0s and vice versa

- Example: the dual of   a (b + c) = a b + a c
  is   a + (bc) = (a + b)(a + c)

- We say that duality holds if both are true.
  (as we will soon see, both are true, so indeed, duality holds for this expression)

Involution



- This theorem states:
  $\overline{\overline{a}} = a$
- Remember that $a\overline{a} = 0$ and $a+\overline{a}=1$.
  - Therefore, $\overline{a}$ is the complement of a and a is also the complement of $\overline{a}$.
  - As the complement of $\overline{a}$ is unique, it follows that $\overline{\overline{a}}=a$.
- Taking the double inverse of a value will give the initial value.

Absorption

- This theorem states:

$$a + ab = a$$

$$a(a+b) = a$$

Absorption

- This theorem states:

  $a + ab = a$ $\qquad\qquad$ $a(a+b) = a$

- To prove the first half of this theorem:

$$\begin{aligned} a + ab &= a \,.\, 1 + ab \\ &= a\,(1 + b) \\ &= a\,(b + 1) \\ &= a\,(1) \\ a + ab &= a \end{aligned}$$

DeMorgan's Theorem

- A key theorem in simplifying Boolean algebra expression is DeMorgan's Theorem. It states:

  $\overline{(a + b)} = \bar{a}\,\bar{b}$ $\overline{(ab)} = \bar{a} + \bar{b}$

DeMorgan's Theorem

- A key theorem in simplifying Boolean algebra expression is DeMorgan's Theorem. It states:

$$\overline{(a + b)} = \bar{a}\,\bar{b} \qquad\qquad \overline{(ab)} = \bar{a} + \bar{b}$$

- Complement the expression $(a(b + z(x + \bar{a}))$ and simplify.

$$\overline{(a(b+z(x + \bar{a})))} \quad = \bar{a} + \overline{(b + z(x + \bar{a}))}$$

$$= \bar{a} + \bar{b}\,\overline{(z(x + \bar{a}))}$$

$$= \bar{a} + \bar{b}(\bar{z} + \overline{(x + a)})$$

$$= \bar{a} + \bar{b}(\bar{z} + \bar{x}\bar{\bar{a}})$$

$$= \bar{a} + \bar{b}(\bar{z} + \bar{x}a)$$

DeMorgan’s Theorem – Exercise

Simplify the expression:

$$\overline{\overline{(x\,\overline{y})}\,(\overline{y} + z)}$$

DeMorgan's Theorem – Exercise

Solution:

$$\overline{\overline{(x\,\overline{y})}\,(\overline{y} + z)}$$

$$\overline{\overline{(X\cdot\overline{Y})}\cdot(\overline{Y}+Z)}$$

$$\overline{\overline{(X\cdot\overline{Y})}}+\overline{(\overline{Y}+Z)}$$

$$(X\cdot\overline{Y})+(\overline{\overline{Y}}\cdot\overline{Z})$$

$$(X\cdot\overline{Y})+(Y\cdot\overline{Z})$$

$$= X\overline{Y}+Y\overline{Z}$$

DeMorgan: exercises

$$z = \overline{\overline{A} + \overline{B} \cdot C}$$

$$\omega = \overline{(A + BC) \cdot (D + EF)}$$

DeMorgan: exercises

$$z = \overline{A + \overline{B} \cdot C}$$

$$= \overline{A} \cdot (\overline{\overline{B} \cdot C})$$

$$= \overline{A} \cdot (\overline{\overline{B}} + \overline{C})$$

$$= \overline{A} \cdot (B + \overline{C})$$

$$\omega = \overline{(A + BC) \cdot (D + EF)}$$

$$= (\overline{A + BC}) + (\overline{D + EF})$$

$$= (\overline{A} \cdot \overline{BC}) + (\overline{D} \cdot \overline{EF})$$

$$= [\overline{A} \cdot (\overline{B} + \overline{C})] + [\overline{D} \cdot (\overline{E} + \overline{F})]$$

$$= \overline{A}\,\overline{B} + \overline{A}\,\overline{C} + \overline{D}\,\overline{E} + \overline{D}\,\overline{F}$$

Boolean Algebra Laws

	Name of Law	Properties
1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A
2.	Commutative Law	A.B = B.A; A+B = B+A
3.	Associative Law	A.(B.C) = A.B.C; A+(B+C) = A+B+C
4.	Idempotent Law	A.A = A; A+A = A
5.	Double Negative Law	A'' = A
6.	Complement Law	A.A' = 0; A+A' = 1
7.	Law of Union	A+1 = A; A+0 = A
8.	De Morgan's Theorem	(AB)' = A'+B'; (A+B)' = A'.B'
9.	Distributive Law	A.(B+C) = (A.B) + (A.C); A+(BC) = (A+B).(A+C)
10.	Absorption Law	A.(A+B) = A; A+(A.B) = A
11.	Common Identities Law	A.(A'+B) = AB; A+(A'B) = A+B

Example 1: Simplify the given Boolean expression.

$$C + (BC)'$$

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$C + (BC)'$ → $\overline{BC}$

Example 1

Simplify the expression: C + (BC)' → $\overline{BC}$

Solution:

C + (BC)'

Rules Used

Apply DeMorgan's Theorem to the (BC)' term

9.	DeMorgan's Theorem	(AB)' = A'+B'; (A+B)' = A'.B'

Example 1

Simplify the expression: C + (BC)' → $\overline{BC}$

Solution:

C + (BC)'

Step 1: C + (B' + C')

Rules Used

9) DeMorgan's Law

9.	DeMorgan's Theorem	(AB)' = A'+B'; (A+B)' = A'.B'

Example 1

Simplify the expression: C + (BC)' → $\overline{BC}$

Solution:

C + (BC)'

Rules Used

Step 1: C + (B' + C') — 8) DeMorgan's Law

Step 2: C + (C' + B') — 2) Commutative Law

2.	Commutative Law	A.B = B.A; A+B = B+A

Example 1

Simplify the expression: C + (BC)’ → $\overline{BC}$

Solution:

C + (BC)’

Rules Used

Step 1: C + (B’ + C’)

8) De Morgan’s Law

Step 2: C + (C’ + B’)

2) Commutative

Step 3: C + C’ + B’

3) Associative Laws

3.	Associative Law	A.(B.C) = A.B.C; A+(B+C) = A+B+C

Example 1

Simplify the expression: C + (BC)’ → $\overline{BC}$

Solution:

C + (BC)’

	C + (BC)’	Rules Used
Step 1:	C + (B’ + C’)	8) DeMorgan’s Law
Step 2:	C + (C’ + B’)	2) Commutative
Step 3:	C + C’ + B’	3) Associative Laws
Step 4:	1 + B’	6) Complement Law

6.	Complementary Law	A.A’ = 0; A+A’ = 1

Example 1

Simplify the expression: C + (BC)' → $\overline{BC}$

Solution:

	C + (BC)'	Rules Used
Step 1:	C + (B' + C')	8) DeMorgan's Law
Step 2:	C + (C' + B')	2) Commutative
Step 3:	C + C' + B'	3) Associative Laws
Step 4:	1 + B'	6) Complement Law
Step 5:	= 1	1) Identity Law

1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A

Example 2: Simplify the given Boolean expression.

$$(AB)'(A'+B)(B'+B)$$

Example 2: Simplify the given Boolean expression.

(AB)'(A'+B)(B'+B)

	Name of Law	Properties
1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A
2.	Commutative Law	A.B = B.A; A+B = B+A
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Example 2: Simplify the given Boolean expression.

$$(AB)'(A'+B)(B'+B)$$

Solution:

$(AB)'(A'+B)(B'+B)$

Rules Used

Step 1: $(AB)'(A'+B)(1)$

6) Complement Law

6.	Complement Law	A.A' = 0; A+A' = 1

Example 2: Simplify the given Boolean expression.

$$(AB)'(A'+B)(B'+B)$$

Solution:

$(AB)'(A'+B)(B'+B)$

Rules Used

Step 1: $(AB)'(A'+B)(1)$ — 6) Complementary Law

Step 2: $(AB)'(A'+B)$ — 1) Identity Law

1.	Identity Law	A+0=A; A+1=1; A.0=0; A.1=A

Example 2: Simplify the given Boolean expression.

(AB)'(A'+B)(B'+B)

Solution:

(AB)'(A'+B)(B'+B)

		Rules Used
Step 1:	(AB)'(A'+B)(1)	6) Complementary Law
Step 2:	(AB)'(A'+B)	1) Identity Law
Step 3:	(A'+B')(A'+B)	8) DeMorgan's Theorem

8.	DeMorgan's Theorem	(AB)' = A'+B'; (A+B)' = A'.B'

Example 2: Simplify the given Boolean expression.

$$(AB)'(A'+B)(B'+B)$$

Solution:

(AB)'(A'+B)(B'+B)

		Rules Used
Step 1:	(AB)'(A'+B)(1)	6) Complementary Law
Step 2:	(AB)'(A'+B)	1) Identity Law
Step 3:	(A'+B')(A'+B)	8) DeMorgan's Theorem
Step 4:	A' + B'B	9) Distributive Law

9.	Distributive Law	A.(B+C) = (A.B) + (A.C); A+(BC) = (A+B).(A+C)

Example 2: Simplify the given Boolean expression.

$$(AB)'(A'+B)(B'+B)$$

Solution:

		Rules Used
	$(AB)'(A'+B)(B'+B)$	
Step 1:	$(AB)'(A'+B)(1)$	6) Complementary Law
Step 2:	$(AB)'(A'+B)$	1) Identity Law
Step 3:	$(A'+B')(A'+B)$	8) DeMorgan's Theorem
Step 4:	$A' + B'B$	9) Distributive Law
Step 5:	$= A'$	6) Complement Law

6.	Complement Law	$A.A' = 0$; $A+A' = 1$

Exercise

Use the Boolean rules to simplify the following expressions:

(a) $X = ABC + \overline{A}B + AB\overline{C}$

(b) $X = \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(c) $AB + \overline{A}C + BC = AB + \overline{A}C$

(d) $(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$

Exercise 6.2

Use the Boolean rules to simplify the following expressions:

(i) $AB + AC + ABC$

(ii) $AB + A(\overline{B} + C) + AB\overline{C}$

(iii) $\overline{A}BC + A\overline{B}C + ABC + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

(iv) $A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C$

Simplify and draw the circuit diagram for

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Simplify and draw the circuit diagram for

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Trick: copy terms (remember: X+X=X)

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}BC + ABC + A\overline{B}C + ABC + AB\overline{C} + ABC$$

Simplify and draw the circuit diagram for

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Trick: ‘duplicate’ terms (remember: X+X=X)

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \boxed{\overline{A}BC + ABC} + \boxed{A\overline{B}C + ABC} + \boxed{AB\overline{C} + ABC}$$

$$= \boxed{BC(\overline{A} + A)} + \boxed{AC(\overline{B} + B)} + \boxed{AB(\overline{C} + C)}$$

Simplify and draw the circuit diagram for

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

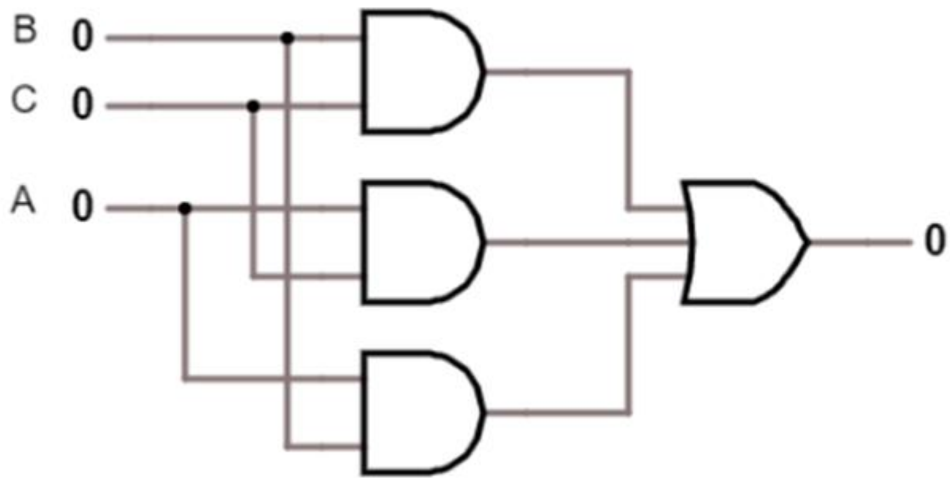
Trick: copy terms (remember: X+X=X)

$$x = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}BC + ABC + A\overline{B}C + ABC + AB\overline{C} + ABC$$

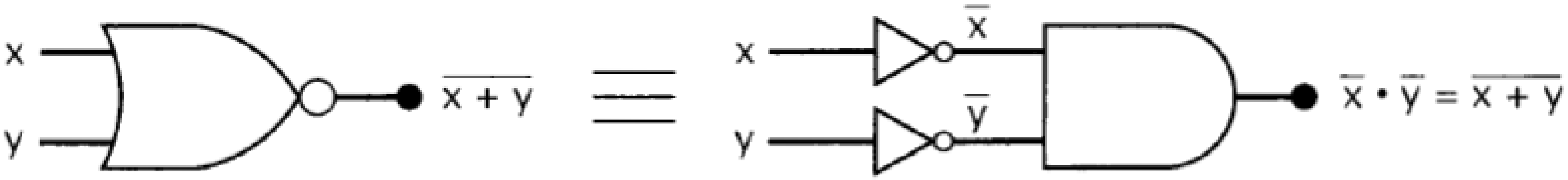
$$= BC(\overline{A} + A) + AC(\overline{B} + B) + AB(\overline{C} + C)$$

$$= BC + AC + AB$$

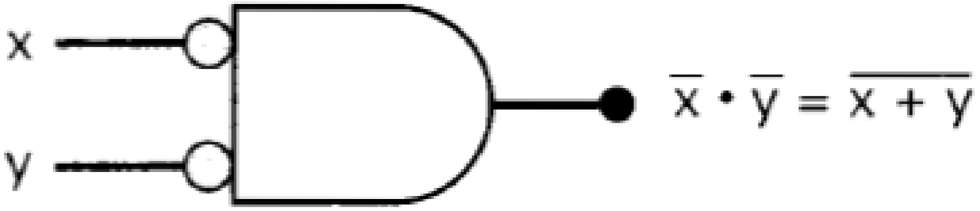


DeMorgan's Theorem and NOR gates

- What does DeMorgan's theorem imply about NOR gates?

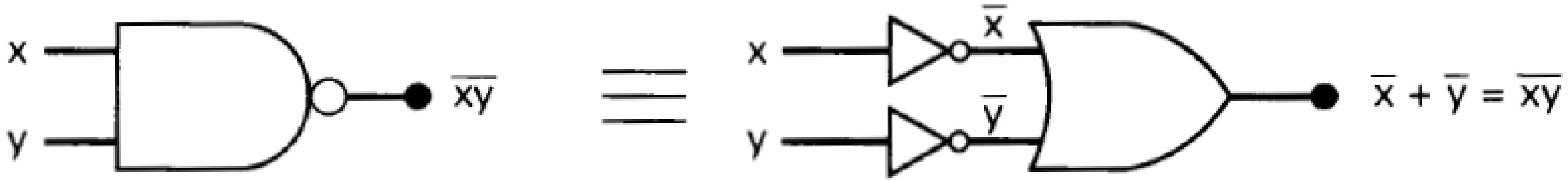


- Another symbol for the NOR function

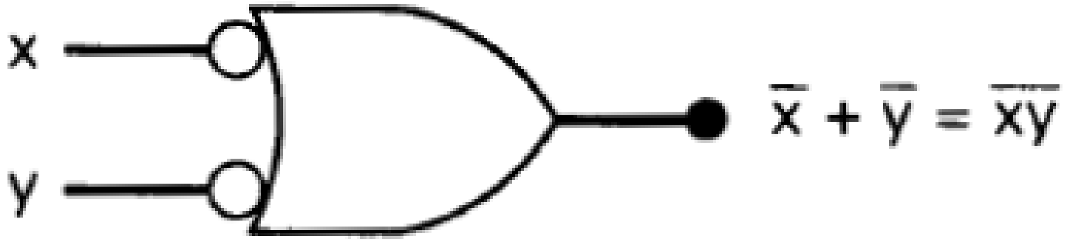


DeMorgan's Theorem and NAND gates

- What does DeMorgan's theorem imply about NAND gates?



- Another symbol for the NAND function



Exclusive OR and Exclusive NOR Circuits

- The exclusive OR, abbreviated XOR produces a HIGH output whenever the two inputs are at opposite levels.
- The exclusive NOR, abbreviated XNOR produces a HIGH output whenever the two inputs are at the same level.
- XOR and XNOR outputs are opposite.

FIGURE 4-20 (a) Exclusive-OR circuit and truth table; (b) traditional XOR gate symbol; (c) IEEE/ANSI symbol for XOR gate.

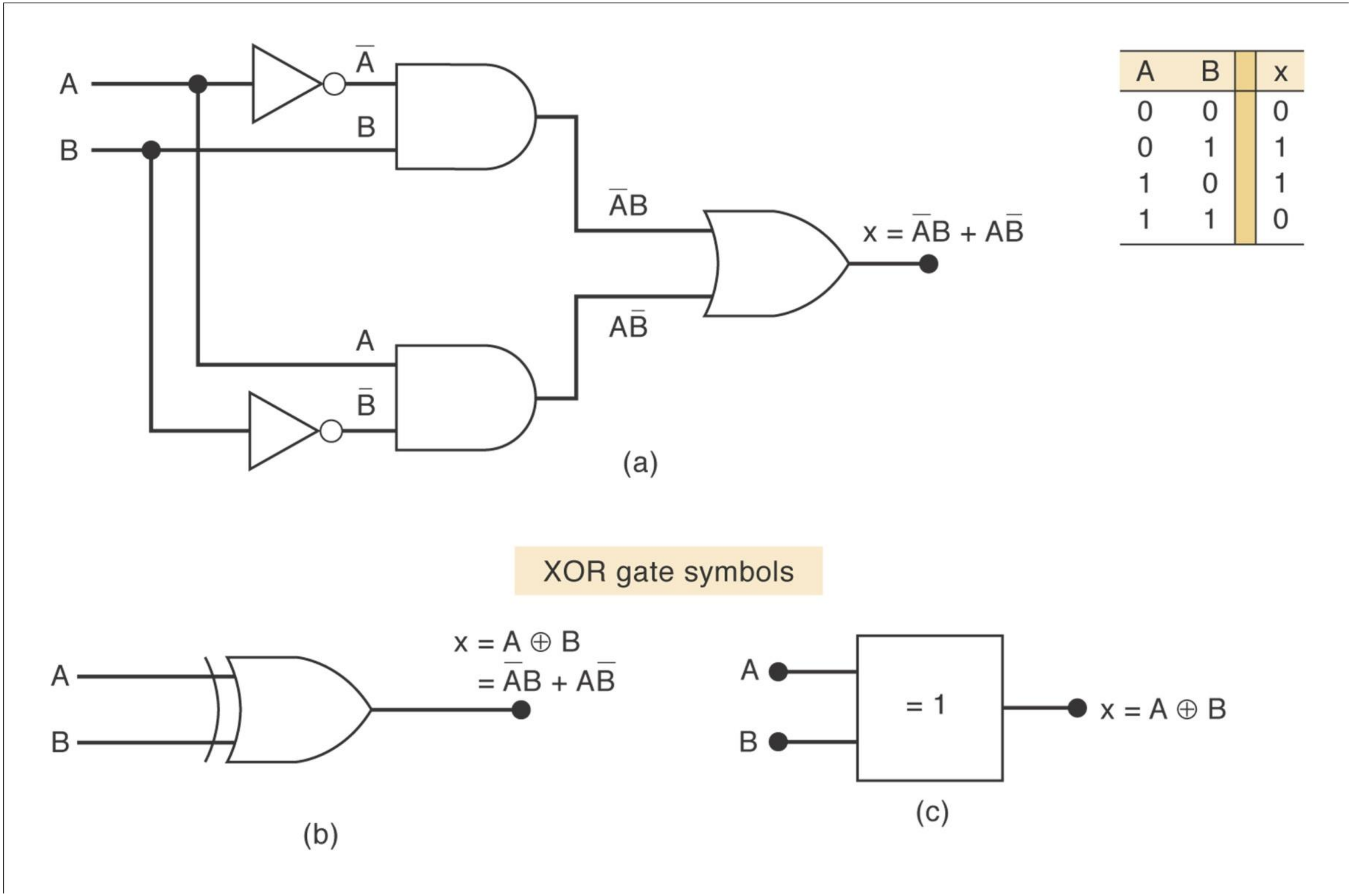
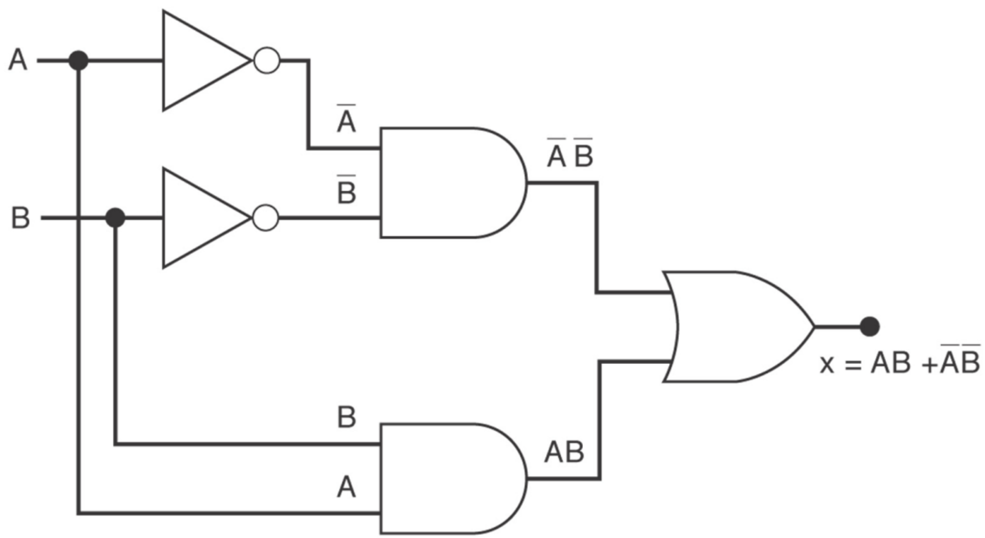
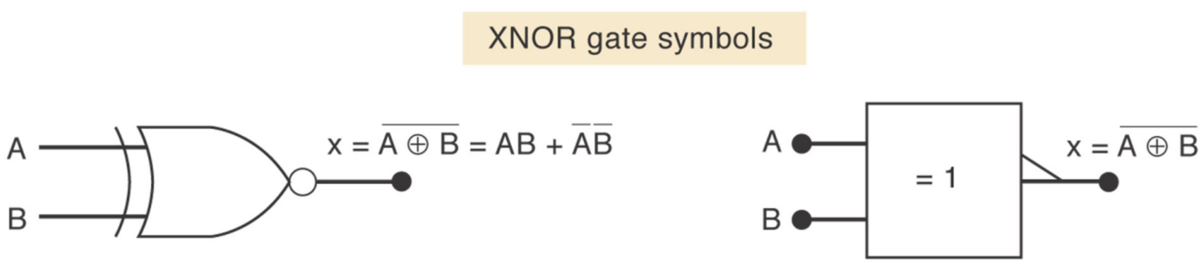


FIGURE 4-21 (a) Exclusive-NOR circuit; (b) traditional symbol for XNOR gate; (c) IEEE/ANSI symbol.



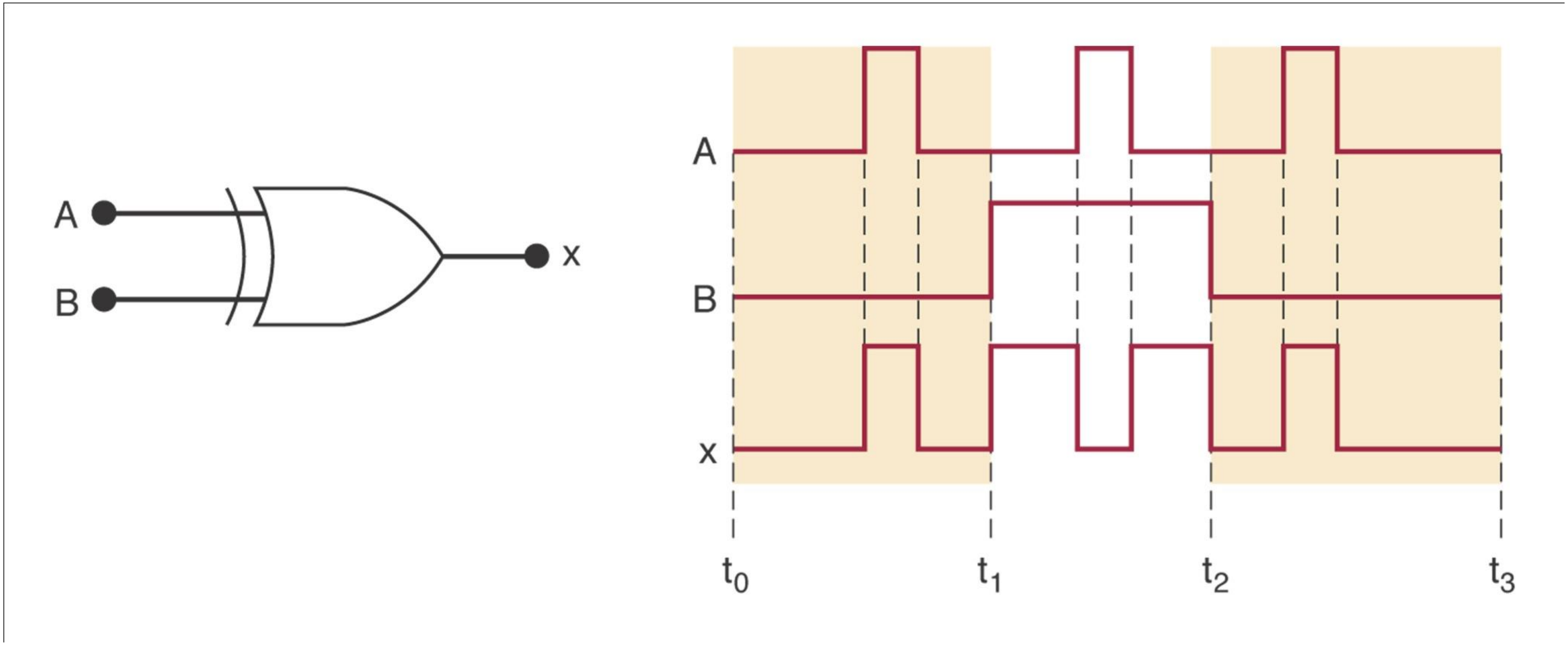
A	B	x
0	0	1
0	1	0
1	0	0
1	1	1

(a)



(b) (c)

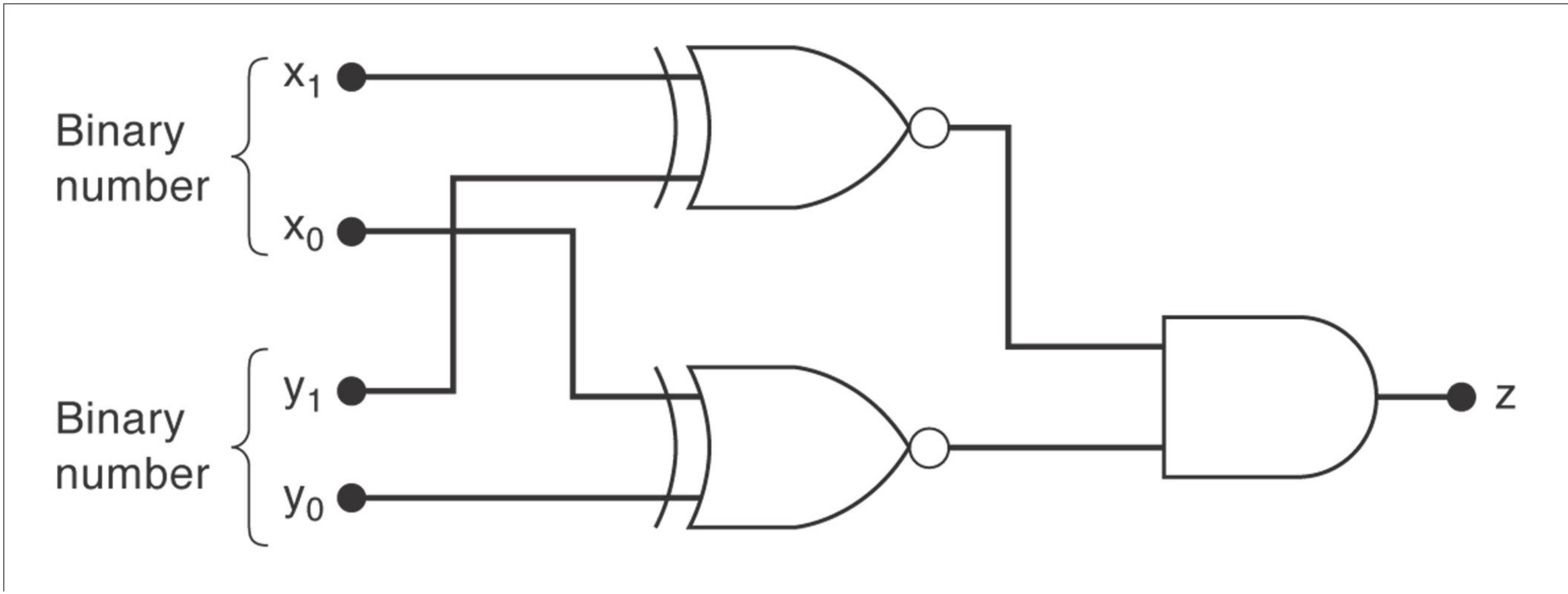
Determine the O/P waveform of the circuit below:



O/P Hi when I/P at different levels

Design a circuit so that the O/P will only be HI when the combination of two sets of two bit binary numbers are equal.

x_1	x_0	y_1	y_0	z (Output)
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1


x1
x0
Binary
number
y1
y0
Binary
number
z

Parity

- XOR and XNOR gates are often used in parity generators and parity checkers
- (more on this in future courses!)