

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
 $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \rightarrow Q \equiv \neg P \vee Q$
4. Commutative laws:
 $P \wedge Q \equiv Q \wedge P$
 $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
 $P \wedge P \equiv P$
 $P \vee P \equiv P$
6. Distributive laws:
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
 $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$
9. Tautology: if \mathbb{T} is a tautology, then
 $P \vee \mathbb{T} \equiv \mathbb{T}$
 $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if \mathbb{F} is a contradiction, then
 $P \vee \mathbb{F} \equiv P$
 $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\frac{P \quad P \rightarrow Q}{Q}$$

- *Modus tollens.*

$$\frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

- *Transitivity.*

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

- *Contrapositive.*

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \rightarrow Q(x))$$

- *Universal* Some P's are Q's

$$\exists x(P(x) \wedge Q(x))$$

- *Negating quantifiers*

$$\neg \forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg \exists x[R(x)] \equiv \forall x[\neg R(x)]$$