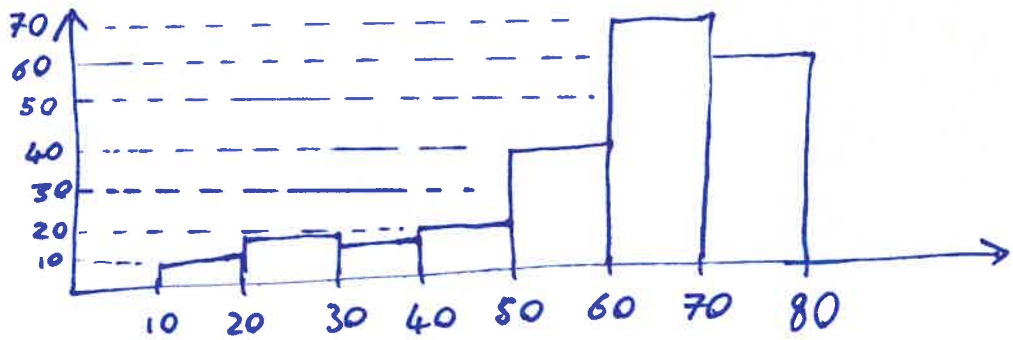
ENGR 123

ASSIGNMENT: TEST REVISION

[1] A set of data has the following histogram.



a) Sketch a CDF that roughly corresponds to this histogram.

b) Sketch a boxplot that roughly corresponds to this histogram.

[2] $X \sim Bin(10, 0.2)$

a) Explain <u>ALL</u> the assumptions behind this probability model.

b) Find $E(X)$, $Var(X)$, $Var(3-4X)$

c) Find $P(X>1)$

[3] X has the following distribution

x	-4	-2	0	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$

a) Find $E(2-X)$

b) Find $P[|X|>1]$

[4] X is a continuous RV with CDF

$$F(x) = \begin{cases} 1 - 1/x^6 & x \geq 1 \\ 0 & x < 0 \end{cases}$$

a) Find $f(x)$

b) Find $E(X)$

c) Find $P(X>3)$

[5] $X \sim N(2, 4)$ Remember $N(\mu, \sigma^2)$ notation.

a) What is $f(x)$

b) What is $E(X)$, $Var(X)$

c) What is $P(X<0)$? $P(X>8)$?

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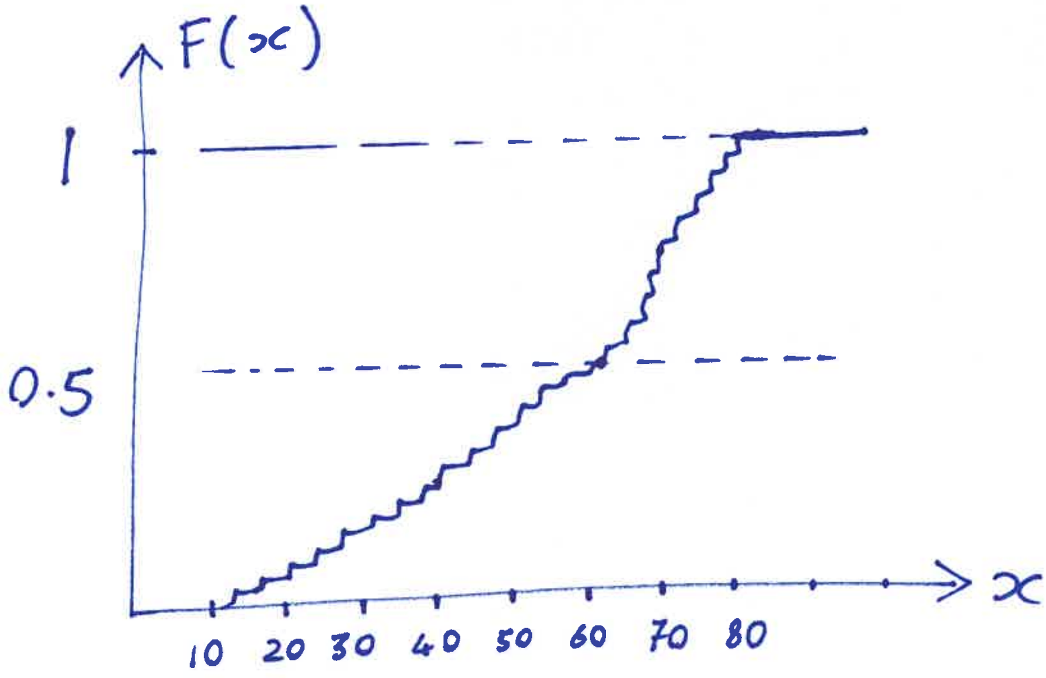
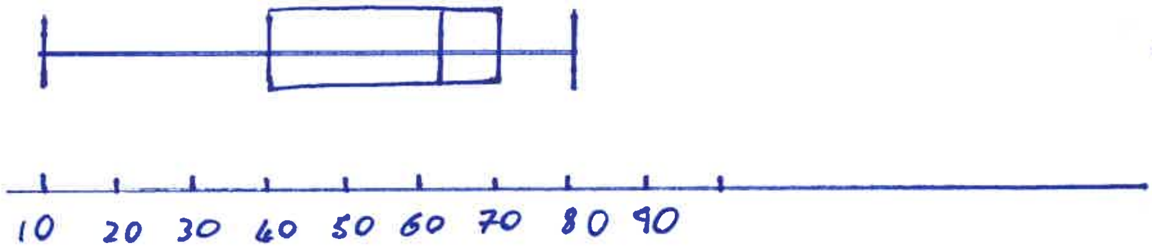
[1] a)



b)



[2] a) X counts the no. of successes in n trials

- fixed no. of trials
- binary outcome
- constant prob. of success
- independent trials

b) $E(X) = np = 10(0.2) = 2$

$Var(X) = np(1-p) = 10 \times 0.2 \times 0.8 = 1.6$

$Var(3-4X) = 16\, Var\, X$

$= 25.6$

c) $P(X>1) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\binom{10}{0} 0.2^0 0.8^{10} + \binom{10}{1} 0.2^1 0.8^9\right]$

$= 1 - [0.8^{10} + 10 \times 0.2 \times 0.8^9]$

$= 1 - [0.10737 + 0.26844]$

$= 0.624$

[3] a) $E(2-X) = 2 - E(X)$

$= 2 - \left[(-4)\frac{1}{4} + (-2)\left(\frac{1}{8}\right) + 0\left(\frac{1}{4}\right) + 2\left(\frac{3}{8}\right)\right]$

$= 2 - \left[-1 - \frac{1}{4} + 6/8\right] = 2 - \left[-\frac{1}{2}\right]$

$= 5/2$

b) $P[|X|>1] = P(X=2 \text{ or } X=-2 \text{ or } X=-4)$

$= 3/8 + 1/8 + 1/4$

$= 6/8 = 3/4$

[4] a) $f(x) = F'(x)$

$$= \begin{cases} \frac{d}{dx} 1 - x^{-6} & x \geq 1 \\ 0 & x < 0 \end{cases}$$

$$= \begin{cases} 6x^{-7} & x \geq 1 \\ 0 & x < 0 \end{cases}$$

b) $E(X) = \int_1^\infty x \times 6x^{-7}\,dx = \int_1^\infty 6x^{-6}\,dx$

$$= \left[\frac{6x^{-5}}{-5}\right]_1^\infty = 0 - \left[\frac{6}{-5}\right]$$

$= 6/5$

c) $P(X>3) = 1-F(3) = 1-\left(1-\frac{1}{36}\right)$

$= \frac{1}{36} = 0.0014$

[5] a)

$$\frac{e^{-\frac{(x-2)^2}{8}}}{\sqrt{2\pi \times 4}} \qquad -\infty < x < \infty$$

b) $E(X) = 2 \quad Var(X) = 4$

c) Using python/matlab/excel or whatever...

$$P(X<0) = 0.1587$$

$$P(X>8) = 0.0013$$

or, by standardizing

$$P(X<0) = P\left(Z < \frac{0-\mu}{\sigma}\right)$$

$$= P(Z < -1) = 0.1587$$

$$P(X>8) = P\left(Z > \frac{8-2}{2}\right)$$

$$= P(Z > 3) = 0.0013$$