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ENGR123 Test Two
50 minutes. 7 questions.
40 marks total
5th October 2018

Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last two pages.

1. Complete the following truth table **[8 marks]**

P	Q	R	$\neg P \leftrightarrow (Q \wedge R)$	$P \rightarrow (Q \rightarrow (\neg R \wedge P))$
0	0	0	1 0 0	1 1 1 0
0	0	1	1 0 0	1 1 0 0
0	1	0	1 0 0	1 0 1 0
0	1	1	1 1 1	1 0 0 0
1	0	0	0 1 0	1 1 1 1
1	0	1	0 1 0	1 1 0 0
1	1	0	0 1 0	1 1 1 1
1	1	1	0 0 1	0 0 0 0
			1 Fin 2	Fin 3 1 2

2. Consider the following jumbled argument:

   (1) All my sons are slim;
   (2) No child of mine is healthy who takes no exercise;
   (3) All gluttons, who are children of mine, are fat;
   (4) No daughter of mine takes any exercise.

   (a) Rewrite each statement using predicates. **[4 marks]**

   (b) Derive a conclusion, and order the statements so that the conclusion follows logically from the premises. **[4 marks]**

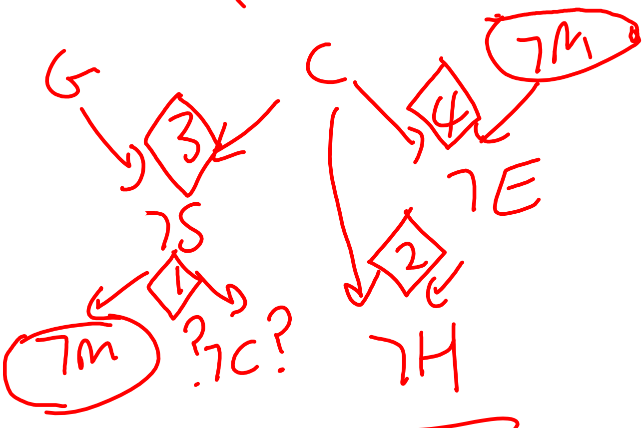
(a) S slim, M male, C child of mine, H healthy
E exercise, G glutton

(1) $\forall x\,([M(x) \wedge C(x)] \rightarrow S(x))$

(2) $\forall y\,([C(y) \wedge \neg E(y)] \rightarrow \neg H(y))$

(3) $\forall z\,([G(z) \wedge C(z)] \rightarrow \neg S(z))$

(4) $\forall u\,([C(u) \wedge \neg M(u)] \rightarrow \neg E(u))$



Visualisation

	Premises
$C \wedge G$	
$\rightarrow \neg S$	(3)
$\rightarrow \neg M \vee \cancel{\neg C}$	Contra (1)
$\rightarrow \neg M$	!
$\rightarrow \neg M \wedge C$	!!
$\rightarrow \neg E$	(4)
$\rightarrow \neg E \wedge C$	!
$\rightarrow \neg H$	Conclusion.

No child of mine who is a glutton, is healthy.

3. Determine the truth values of the following statements, where the variables are people, and happy(a, b) means that they can live happily together. Provide a brief explanation in each case. **[4 marks]**

   (a) $\forall n\, \exists m$ happy(m, n)

   (b) $\exists m\, \forall n$ happy(m, n)

(a) Everyone has someone who they live happily with. would like this to be TRUE

(b) Someone lives happily with everyone. a saint they'd be, maybe FALSE.

4. What is the negation of $\exists m\, \forall n$ happy(m, n)? **[2 marks]**

$\forall m\, \exists n\, \neg$happy$(m,n)$

Everyone has someone they won't happily live with.

5. (a) What properties must a relation satisfy to be an equivalence relation? **[3 marks]**

(b) Let $R \subset \mathbb{Z} \times \mathbb{Z}$ be the relation on integers given by

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + y \text{ is even}\}$$

i. Is R a partial order? Explain why or why not. **[2 marks]**

ii. Is R a function? Explain why or why not. **[2 marks]**

(a) Reflexive, Symmetric, Transitive

(b i) R is reflexive, $a+a=2a$ is always even.

R is symmetric, if $a+b=2k$ then $b+a=2k$ as well!

R is transitive, if $a+b=2k$ and $b+c=2l$, then

$$(a+b)+(b+c)=2k+2l$$
$$a+2b+c=2k+2l$$
$$a+c=2k+2l-2b$$
$$=2(k+l-b)$$

So $a+c$ is even!

6. Prove by contraposition that if the product of two natural numbers is greater than 100, then at least one of them is greater than or equal to 10. **[5 marks]**

Suppose both natural numbers are not greater than or equal to 10.

Then $x < 10$ and $y < 10$.

So $x \cdot y < 10 \times 10$
$= 100$

Hence the product is less than 100.

So the product is <u>not</u> greater than or equal to 100.

7. Prove by contradiction that the sum of two positive numbers is positive. **[5 marks]**

Suppose $x > 0$, $y > 0$ and $x + y \not> 0$

Then $x + y > 0 + 0$
$= 0$

but $x + y \not> 0$

Contradiction!

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
   $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
   $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \to Q \equiv \neg P \vee Q$
4. Commutative laws:
   $P \wedge Q \equiv Q \wedge P$
   $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
   $P \wedge P \equiv P$
   $P \vee P \equiv P$
6. Distributive laws:
   $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
   $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
   $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
   $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \to Q) \equiv (\neg Q \to \neg P)$
9. Tautology: if $\mathbb{T}$ is a tautology, then
   $P \vee \mathbb{T} \equiv \mathbb{T}$
   $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if $\mathbb{F}$ is a contradiction, then
    $P \vee \mathbb{F} \equiv P$
    $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\begin{array}{l} P \\ P \to Q \\ \hline Q \end{array}$$

- *Modus tollens.*

$$\begin{array}{l} P \to Q \\ \neg Q \\ \hline \neg P \end{array}$$

- *Or-elimination.*

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline Q \end{array}$$

- *And-elimination.*

$$\begin{array}{l} P \wedge Q \\ \hline P \end{array}$$

- *Transitivity.*

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$

- *Or-introduction.*

$$\begin{array}{l} P \\ \hline P \vee Q \end{array}$$

- *Contrapositive.*

$$\begin{array}{l} P \to Q \\ \hline \neg Q \to \neg P \end{array}$$

- *Implies-introduction.*

$$\begin{array}{l} Q \\ \hline P \to Q \end{array}$$