**ENGR123 Test One**
50 minutes. 7 questions.
**40 marks total**
6 September 2016

**Name:**

**ID Number:**

Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last page.

1. Complete the following truth table                                   [**8 marks**]

| $P$ | $Q$ | $R$ | $\neg P \leftrightarrow (Q \wedge R)$ | $P \to (Q \to (\neg R \wedge P))$ |
|---|---|---|---|---|
| 0 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 0 | 1 | 0 | | |
| 0 | 1 | 1 | | |
| 1 | 0 | 0 | | |
| 1 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

2. Consider the following jumbled argument:

No birds, except ostriches, are nine feet high.
The birds in the aviary do not eat mince pies.
There are no birds in this aviary that belong to anyone but me.
No ostrich eats mince pies.
I have no birds less than nine feet high.

(a) Rewrite each statement using predicates. [4 marks]

(b) Identify the conclusion, and order the statements so that the conclusion follows logically from the premises. [4 marks]

3. Determine the truth values of the following statements (where the variables are *integers*). Provide a brief explanation in each case.    [**4 marks**]

(a) $\forall n\ \exists m\ (n^2 \geq m)$

(b) $\exists m\ \forall n\ (n^2 \geq m)$

4. What is the negation of $\exists m\ \forall n\ (n^2 \geq m)$?    [**2 marks**]

5. (a) What properties must a relation satisfy to be an equivalence relation?    **[3 marks]**

   (b) Let $R \subset \mathbb{Z} \times \mathbb{Z}$ be the relation on integers given by

   $$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + y \text{ is a multiple of } 5\}$$

      i. Is $R$ a partial order? Explain why or why not.    **[2 marks]**

      ii. Is $R$ a function? Explain why or why not.    **[2 marks]**